



Multi-criteria decision making approach based on PROMETHEE with probabilistic simplified neutrosophic sets

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Abstract

Probabilistic simplified neutrosophic set (*PSNS*) is an important tool to describe the vagueness existing in the real life. In this study, we define a *PSNS* and discuss some of theoretical set operations of *PSNS*s. Also we propose the concepts of module on *PSNS*s, as well as inner product and projection operator between two *PSNS*s. In relation to this new set, we introduce a probabilistic simplified neutrosophic number (*PSNN*). A *PSNN* has three components which is called probabilistic-valued truth membership degree, probabilistic-valued indeterminacy membership degree and probabilistic-valued falsity membership degree, respectively. We give some of algebraic operational rules, a score function and an accuracy function on *PSNN*s. Furthermore, we introduce two aggregation operators called the probabilistic simplified neutrosophic weighted arithmetic average operator and the probabilistic simplified weighted geometric average operator on *PSNN*s. Furthermore, we determine weights of criteria with a method that is based on fuzzy measure and develop a method based on preference function to determine weight of each decision maker. We present an extended PROMETHEE method based on *PSNS*s for group decision problems. Finally, as an application of this theory, we give a practice on a multi-criteria group decision making problem based on *PSNN*s by using extended PROMETHEE method to ensure stability of the proposed method.

Keywords Probabilistic simplified neutrosophic set · Group decision making · PROMETHEE method · Projection measure

1 Introduction

Multi-criteria decision making methods (MCDMs) are applied for different fields by many researchers. But it is difficult for decision makers to model problems with uncertain, incomplete information. To overcome this

situation and model uncertain information, Zadeh (1965) defined fuzzy sets (FSs). A FS is characterized with its membership information. There are many papers in which FSs are utilized as a model in decision problems, pattern recognition and approximate reasoning. But FSs are insufficient to express certain situations because of the lack of the information of uncertainty. With this mind, Atanassov (1986) developed the concept of intuition fuzzy set (IFS). Unlike FSs, we have non-membership information and the sum of the membership and non-membership is less than or equal to one. Similarly, it has many application areas, including decision problems (Liu and Wang 2007), medical diagnosis (Shinoj and John 2012), neural networks (Sotirov et al. 2009; Pap 1997), etc.

But there are different types of uncertainty in our life. For example, when an expert is asked to think about a situation, he/she can say that the truth membership degree is 0.5, the indeterminacy membership is 0.3 and the falsity membership degree is 0.7. This situation cannot be explained with the sets defined above. Based on this idea, Smarandache (1999) developed neutrosophic logic and neutrosophic set

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(NS). The NSs are characterized with three memberships called truth membership, indeterminacy membership and falsity membership. The sum of these memberships is less than or equal to three. Here, the membership degrees take values in non-standard unit interval $]0^-, 1^+[$ which is the generalization of standard unit interval $[0, 1]$. That is, NSs have a philosophical mean. So, to apply practical examples, many special cases of NSs are developed. For example, a single-valued neutrosophic sets (SVNS) and an interval neutrosophic set (IVNS) are introduced in Wang et al. (2004, 2005), respectively. Broumi et al. (2016a, b) studied SVNS. Zhang et al. (2014) proposed a method for multi-criteria decision problems with IVNS, and Zhang et al. (2016) gave some applications for MCDM problems under interval-valued neutrosophic environment. Zhang et al. (2015) developed correlation coefficient for INSSs. Also Ye (2014a) proposed simplified neutrosophic set (SNS) and gave some operational rules and subsequently applied it to MCDM problems. Peng et al. (2016) modified some of operational rules given in Ye (2014a). There are many application areas, including these sets: Ye and Zhang (2014) and Ye (2014b) studied similarity measure with SVNS and IVNS and Ye (2013) developed a method for decision making problems using correlation coefficient of SVNS. Majumdar and Samanta (2014) defined similarity and entropy measures for NSs. Peng et al. (2014) introduced a new approach to MCDM problems with simplified neutrosophic information. Biswas et al. (2016) developed a method based on TOPSIS for multi-criteria decision problems with simplified neutrosophic environment.

Aggregation is a function which takes the several numerical values to a single value, and an aggregation function performs this operation (Tian et al. 2017). Grabisch et al. (2009) defined two simplified neutrosophic linguistic power aggregation operators in the form of simplified neutrosophic linguistic numbers. Mohamed et al. (2017) proposed some linear programming problems based on neutrosophic environment. Recently, studies on NSs rapidly have been increased in solving uncertain problems (Şahin 2017; Şahin and Liu 2017a; Bolturk and Kahraman 2018; Ye 2018; Karaşan and Kahraman 2018; Şahin 2019; Abdel-Basset et al. 2018).

Most decision problems, including economic, industrial and financial, have many criteria that decision makers have to consider. Thus, the structure of the decision problems is complex because of the number of criteria. So making the optimal choice is difficult and complicated, and thus, various methods are developed. TOPSIS (Hwang and Yoon 1981), VIKOR (Opricovic and Tzeng 2002) and ELECTRE (Benayoun et al. 1969) methods are part of them. Mareschal and Vincke (1984) developed PROMETHEE method for decision problems. In decision problems

examined with PROMETHEE, preference function of each criterion is determined and alternatives are evaluated with respect to the preference functions.

In decision making process, decision makers may be hesitant to present their preference information (Dubois et al. 2000) and may need knowledge of possibility about each neutrosophic element. Recently, Şahin and Liu (2017b) introduced possibility-induced simplified neutrosophic number (PISNN) which partially solves this problem. A PISNN assigns a probability value to the neutrosophic number. But when decision makers express their evaluations with neutrosophic elements, they may hesitate about each of neutrosophic component. This idea requires a new set theory. For this reason, we define a new set called probabilistic simplified neutrosophic set (PSNS) and introduce some of the theoretical set properties. Taking into consideration components and their possibilities at the same time, it is more effective and reliable to describe problems emerging in many fields. Also we introduce some aggregation operators such as probabilistic simplified neutrosophic weighted arithmetic average (PSNWA) operator and probabilistic simplified neutrosophic weighted geometric average (PSNWG) operator. Furthermore, we determine weights of criteria with a proposed method which is based on fuzzy measure and develop a method based on preference function to determine weight of each decision maker. Finally, based on an extended PROMETHEE method, we propose a multi-criteria decision making method with probabilistic simplified neutrosophic information. To observe the effectiveness and reliability of the proposed method, an energy source problem is considered and results are compared with other methods under neutrosophic environment.

Through the paper, first we give some backgrounds existing in the literature. Then, in Sect. 2, some basic concepts are investigated. In Sect. 3, we introduce a new set theory called the probabilistic simplified neutrosophic set theory (PSNS). We give some operational rules of PSNS. Also we define the concepts such as distance measure, inner product, module and projection operator related to this new set. In Sect. 4, we introduce two aggregation operators called probabilistic simplified neutrosophic weighted arithmetic average (PSNWA) operator and probabilistic simplified neutrosophic weighted geometric average (PSNWG) operator and give some properties of them. In Sect. 5, we develop a new method to determine weight of decision makers based on preference function. Also a method to determine weight of criteria is improved based on fuzzy measure and steps of the classical PROMETHEE method are summarized. Then, we developed an extended PROMETHEE method. In Sect. 6, we give a practice on solving a renewable energy source problem to demonstrate the effectiveness and stability of the proposed method. In Sect. 7, we make a comparison with other methods, and finally, in Sect. 8, conclusions about the paper are given.

2 Preliminaries

In this section, some basic concepts and definitions which will be used in this paper are introduced.

Definition 2.1 (Smarandache 1999) Let X be a space of points (objects) with a generic element in X denoted by x . A neutrosophic set A in X is characterized by a truth membership function T_A , an indeterminacy membership I_A and a falsity membership function F_A . $T_A(x)$, $I_A(x)$ and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$.

There is no restriction on the sum of $T_A(x)$, $I_A(x)$ and $F_A(x)$, so $0^- \leq \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \leq 3^+$

Definition 2.2 (Ye 2014a) Let X be a space of points (objects), with a generic element in X , denoted by x . A NS A is characterized by $T_A(x)$, $I_A(x)$ and $F_A(x)$, which are subintervals/subsets in the standard interval $[0, 1]$, that is $T_A : X \rightarrow [0, 1]$, $I_A : X \rightarrow [0, 1]$ and $F_A : X \rightarrow [0, 1]$. Then, the A is denoted by

$$A = \{ \langle x; T_A(x), I_A(x), F_A(x) \rangle : x \in X \}$$

which is called an simplified neutrosophic set (SNS).

If X has only one element, $A = \langle T_A(x), I_A(x), F_A(x) \rangle$ is called a *SNN*. Shortly, a *SNN* is denoted by $A = \langle T_A, I_A, F_A \rangle$.

Peng et al. (2016) introduced some operational rules of *SNNs* as follows:

Definition 2.3 (Peng et al. 2016) Let $A = \langle T_A, I_A, F_A \rangle$ and $B = \langle T_B, I_B, F_B \rangle$ be two *SNNs* and $\lambda > 0$. Then,

- (i) $A \oplus B = \langle T_A + T_B - T_A T_B, I_A I_B, F_A F_B \rangle$;
- (ii) $A \otimes B = \langle T_A T_B, I_A + I_B - I_A I_B, F_A + F_B - F_A F_B \rangle$;
- (iii) $\lambda A = \langle 1 - (1 - T_A)^\lambda, I_A^\lambda, F_A^\lambda \rangle$;
- (iv) $A^\lambda = \langle T_A^\lambda, 1 - (1 - I_A)^\lambda, 1 - (1 - F_A)^\lambda \rangle$

(2.1)

Next, we introduce *PSNS* which is based on the idea that each component of an *SNN* has a probability.

3 Probabilistic simplified neutrosophic set (PSNS)

Definition 3.1 Let X be a universe of discourse. A *PSNS* is defined as follows:

$$K = \{ \langle x; T_K(x)(P_{T_K(x)}), I_K(x)(P_{I_K(x)}), F_K(x)(P_{F_K(x)}) \rangle : x \in X \}$$

where $0 < P_{T_K(x)}, P_{I_K(x)}, P_{F_K(x)} \leq 1$, $T_K(x) \in [0, 1]$, $I_K(x) \in [0, 1]$ and $F_K(x) \in [0, 1]$ with $0 \leq T_K(x) + I_K(x) + F_K(x) \leq 3$ for any $x \in X$.

Obviously, a *PSNS* consists of its simplified neutrosophic components and their possibility values.

We introduce some of operational set rules on *PSNSs*.

Definition 3.2 Let $X \neq \emptyset$ and A, A_1 and A_2 be three *PSNSs* as follows:

$$\begin{aligned} A &= \{ \langle x; T_A(x)(P_{T_A(x)}), I_A(x)(P_{I_A(x)}), F_A(x)(P_{F_A(x)}) \rangle : x \in X \}, \\ A_1 &= \{ \langle x; T_{A_1}(x)(P_{T_{A_1}(x)}), I_{A_1}(x)(P_{I_{A_1}(x)}), F_{A_1}(x)(P_{F_{A_1}(x)}) \rangle : x \in X \}, \\ A_2 &= \{ \langle x; T_{A_2}(x)(P_{T_{A_2}(x)}), I_{A_2}(x)(P_{I_{A_2}(x)}), F_{A_2}(x)(P_{F_{A_2}(x)}) \rangle : x \in X \} \end{aligned}$$

Then, we define the following properties:

- (i) $A_1 \subseteq A_2 \Leftrightarrow T_{A_1}(x) \leq T_{A_2}(x), P_{T_{A_1}(x)} \leq P_{T_{A_2}(x)};$
 $I_{A_1}(x) \geq I_{A_2}(x), P_{I_{A_1}(x)} \geq P_{I_{A_2}(x)};$
 $F_{A_1}(x) \geq F_{A_2}(x), P_{F_{A_1}(x)} \geq P_{F_{A_2}(x)}$ for $x \in X$
- (ii) $A^C = \{ \langle x; F_A(x)(P_{F_A(x)}), (1 - I_A(x))(1 - P_{I_A(x)}), T_A(x)(P_{T_A(x)}) \rangle : x \in X \}$

where A^C denotes the complement of A

$$\begin{aligned} \text{(iii)} \quad A_1 \cap A_2 &= \left\{ \langle x; \min\{T_{A_1}(x), T_{A_2}(x)\} \left(\min\{P_{T_{A_1}(x)}, P_{T_{A_2}(x)}\} \right), \right. \\ &\quad \left. \max\{I_{A_1}(x), I_{A_2}(x)\} \left(\max\{P_{I_{A_1}(x)}, P_{I_{A_2}(x)}\} \right), \right. \\ &\quad \left. \max\{F_{A_1}(x), F_{A_2}(x)\} \left(\max\{P_{F_{A_1}(x)}, P_{F_{A_2}(x)}\} \right) \rangle : x \in X \right\} \\ \text{(iv)} \quad A_1 \cup A_2 &= \left\{ \langle x; \max\{T_{A_1}(x), T_{A_2}(x)\} \left(\max\{P_{T_{A_1}(x)}, P_{T_{A_2}(x)}\} \right), \right. \\ &\quad \left. \min\{I_{A_1}(x), I_{A_2}(x)\} \left(\min\{P_{I_{A_1}(x)}, P_{I_{A_2}(x)}\} \right), \right. \\ &\quad \left. \min\{F_{A_1}(x), F_{A_2}(x)\} \left(\min\{P_{F_{A_1}(x)}, P_{F_{A_2}(x)}\} \right) \rangle : x \in X \right\} \end{aligned}$$

For convenience, $\alpha = (T(P_T), I(P_I), F(P_F))$ is defined as a probabilistic simplified neutrosophic number (*PSNN*), and *PSNN*(X) denotes all of *PSNNs* on X .

Now, we give some operational laws for *PSNNs*.

Definition 3.3 Let $\alpha_1 = (T_1(P_{T_1}), I_1(P_{I_1}), F_1(P_{F_1}))$ and $\alpha_2 = (T_2(P_{T_2}), I_2(P_{I_2}), F_2(P_{F_2}))$ be two *PSNNs* and $\lambda > 0$, then

- (i) $\alpha_1 \oplus \alpha_2 = \left(T_1 + T_2 - T_1 T_2 \left(2! \frac{P_{T_1} P_{T_2}}{P_{T_1} + P_{T_2}} \right), \right.$
 $I_1 I_2 \left(2! \frac{P_{I_1} P_{I_2}}{P_{I_1} + P_{I_2}} \right), F_1 F_2 \left(2! \frac{P_{F_1} P_{F_2}}{P_{F_1} + P_{F_2}} \right) \Bigg);$
- (ii) $\alpha_1 \otimes \alpha_2 = \left(T_1 T_2 \left(2! \frac{P_{T_1} P_{T_2}}{P_{T_1} + P_{T_2}} \right), I_1 + I_2 - I_1 I_2 \right.$
 $\left(2! \frac{P_{I_1} P_{I_2}}{P_{I_1} + P_{I_2}} \right), F_1 + F_2 - F_1 F_2 \left(2! \frac{P_{F_1} P_{F_2}}{P_{F_1} + P_{F_2}} \right) \Bigg);$
- (iii) $\lambda \alpha_1 = \left(1 - (1 - T_1)^\lambda (P_{T_1}), I_1^\lambda (P_{I_1}), F_1^\lambda (P_{F_1}) \right);$
- (iv) $\alpha_1^\lambda = \left(T_1^\lambda (P_{T_1}), 1 - (1 - I_1)^\lambda (P_{I_1}), 1 - (1 - F_1)^\lambda (P_{F_1}) \right).$

(3.1)

Theorem 3.4 Let $\lambda, \lambda_1, \lambda_2 > 0$ and $\alpha, \alpha_1, \alpha_2$ be three PSNNs. Then,

$$\begin{aligned}
 (1) \quad & \alpha_1 \oplus \alpha_2 = \alpha_2 \oplus \alpha_1, \\
 (2) \quad & \alpha_1 \otimes \alpha_2 = \alpha_2 \otimes \alpha_1, \\
 (3) \quad & \lambda(\alpha_1 \oplus \alpha_2) = \lambda\alpha_1 \oplus \lambda\alpha_2, \\
 (4) \quad & (\alpha_1 \otimes \alpha_2)^\lambda = \alpha_1^\lambda \otimes \alpha_2^\lambda, \\
 (5) \quad & \lambda_1\alpha \oplus \lambda_2\alpha = (\lambda_1 + \lambda_2)\alpha, \\
 (6) \quad & \alpha^{\lambda_1} \otimes \alpha^{\lambda_2} = \alpha^{\lambda_1 + \lambda_2} \\
 (7) \quad & (\alpha^{\lambda_1})^{\lambda_2} = \alpha^{\lambda_1\lambda_2}.
 \end{aligned} \tag{3.2}$$

Proof

1. From Definition 3.3 (i), we have

$$\begin{aligned}
 \alpha_1 \oplus \alpha_2 &= \left((T_{\alpha_1} + T_{\alpha_2} - T_{\alpha_1}T_{\alpha_2}) \left(2! \frac{P_{T_{\alpha_1}}P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), \right. \\
 &\quad \left. I_{\alpha_1}I_{\alpha_2} \left(2! \frac{P_{I_{\alpha_1}}P_{I_{\alpha_2}}}{P_{I_{\alpha_1}} + P_{I_{\alpha_2}}} \right), F_{\alpha_1}F_{\alpha_2} \left(2! \frac{P_{F_{\alpha_1}}P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \right) \\
 &= \left((T_{\alpha_2} + T_{\alpha_1} - T_{\alpha_2}T_{\alpha_1}) \left(2! \frac{P_{T_{\alpha_2}}P_{T_{\alpha_1}}}{P_{T_{\alpha_2}} + P_{T_{\alpha_1}}} \right), \right. \\
 &\quad \left. I_{\alpha_2}I_{\alpha_1} \left(2! \frac{P_{I_{\alpha_2}}P_{I_{\alpha_1}}}{P_{I_{\alpha_2}} + P_{I_{\alpha_1}}} \right), F_{\alpha_1}F_{\alpha_2} \left(2! \frac{P_{F_{\alpha_2}}P_{F_{\alpha_1}}}{P_{F_{\alpha_2}} + P_{F_{\alpha_1}}} \right) \right) \\
 &= \alpha_2 \oplus \alpha_1
 \end{aligned}$$

2. From Definition 3.3 (ii), we have

$$\begin{aligned}
 \alpha_1 \otimes \alpha_2 &= \left(T_{\alpha_1}T_{\alpha_2} \left(2! \frac{P_{T_{\alpha_1}}P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), (I_{\alpha_1} + I_{\alpha_2} - I_{\alpha_1}I_{\alpha_2}) \right. \\
 &\quad \left. \left(2! \frac{P_{I_{\alpha_1}}P_{I_{\alpha_2}}}{P_{I_{\alpha_1}} + P_{I_{\alpha_2}}} \right), (F_{\alpha_1} + F_{\alpha_2} - F_{\alpha_1}F_{\alpha_2}) \left(2! \frac{P_{F_{\alpha_1}}P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \right) \\
 &= \left(T_{\alpha_2}T_{\alpha_1} \left(2! \frac{P_{T_{\alpha_2}}P_{T_{\alpha_1}}}{P_{T_{\alpha_2}} + P_{T_{\alpha_1}}} \right), (I_{\alpha_2} + I_{\alpha_1} - I_{\alpha_2}I_{\alpha_1}) \right. \\
 &\quad \left. \left(2! \frac{P_{I_{\alpha_2}}P_{I_{\alpha_1}}}{P_{I_{\alpha_2}} + P_{I_{\alpha_1}}} \right), (F_{\alpha_2} + F_{\alpha_1} - F_{\alpha_2}F_{\alpha_1}) \left(2! \frac{P_{F_{\alpha_2}}P_{F_{\alpha_1}}}{P_{F_{\alpha_2}} + P_{F_{\alpha_1}}} \right) \right) \\
 &= \alpha_2 \otimes \alpha_1
 \end{aligned}$$

3. From Definition 3.3 (i) and (iii), we have

$$\begin{aligned}
 \lambda(\alpha_1 \oplus \alpha_2) &= \left(\left(1 - (1 - (T_{\alpha_1} + T_{\alpha_2} - T_{\alpha_1}T_{\alpha_2}))^\lambda \right) \right. \\
 &\quad \left. \left(2! \frac{P_{T_{\alpha_1}}P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), T_{\alpha_1}^\lambda T_{\alpha_2}^\lambda \left(2! \frac{P_{I_{\alpha_1}}P_{I_{\alpha_2}}}{P_{I_{\alpha_1}} + P_{I_{\alpha_2}}} \right), F_{\alpha_1}^\lambda F_{\alpha_2}^\lambda \left(2! \frac{P_{F_{\alpha_1}}P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \right) \\
 &= \left(\left(1 - (1 - T_{\alpha_1})^\lambda (1 - T_{\alpha_2})^\lambda \right) \right. \\
 &\quad \left. \left(2! \frac{P_{T_{\alpha_1}}P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), I_{\alpha_1}^\lambda I_{\alpha_2}^\lambda \left(2! \frac{P_{I_{\alpha_1}}P_{I_{\alpha_2}}}{P_{I_{\alpha_1}} + P_{I_{\alpha_2}}} \right), F_{\alpha_1}^\lambda F_{\alpha_2}^\lambda \left(2! \frac{P_{F_{\alpha_1}}P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \right)
 \end{aligned} \tag{3.3}$$

On the other hand, we have

$$\begin{aligned}
 \lambda\alpha_1 &= \left((1 - (1 - T_{\alpha_1})^\lambda) (P_{T_{\alpha_1}}), I_{\alpha_1}^\lambda (P_{I_{\alpha_1}}), F_{\alpha_1}^\lambda (P_{F_{\alpha_1}}) \right) \\
 \lambda\alpha_2 &= \left((1 - (1 - T_{\alpha_2})^\lambda) (P_{T_{\alpha_2}}), I_{\alpha_2}^\lambda (P_{I_{\alpha_2}}), F_{\alpha_2}^\lambda (P_{F_{\alpha_2}}) \right)
 \end{aligned}$$

and

$$\begin{aligned}
 \lambda\alpha_1 + \lambda\alpha_2 &= \left(\left(1 - (1 - T_{\alpha_1})^\lambda + 1 - (1 - T_{\alpha_2})^\lambda - (1 - (1 - T_{\alpha_1})^\lambda) (1 - (1 - T_{\alpha_2})^\lambda) \right) \left(2! \frac{P_{T_{\alpha_1}}P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), \right. \\
 &\quad \left. I_{\alpha_1}^\lambda I_{\alpha_2}^\lambda \left(2! \frac{P_{T_{\alpha_1}}P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), F_{\alpha_1}^\lambda F_{\alpha_2}^\lambda \left(2! \frac{P_{F_{\alpha_1}}P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \right) \\
 &= \left(\left(1 - (1 - T_{\alpha_1})^\lambda (1 - T_{\alpha_2})^\lambda \right) \left(2! \frac{P_{T_{\alpha_1}}P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), F_{\alpha_1}^\lambda F_{\alpha_2}^\lambda \left(2! \frac{P_{F_{\alpha_1}}P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \right) \\
 &\quad \left(I_{\alpha_1}^\lambda I_{\alpha_2}^\lambda \left(2! \frac{P_{T_{\alpha_1}}P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), \right.
 \end{aligned} \tag{3.4}$$

From Eqs. (3.3) and (3.4),

$$\lambda(\alpha_1 \oplus \alpha_2) = \lambda\alpha_1 \oplus \lambda\alpha_2$$

is obtained.

4. From Definition 3.3 (ii) and (iv), we have

$$\begin{aligned}
(\alpha_1 \otimes \alpha_2)^\lambda &= \left(T_{\alpha_1}^\lambda T_{\alpha_2}^\lambda \left(2! \frac{P_{T_{\alpha_1}} P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), \right. \\
&\quad \left(1 - (1 - (I_{\alpha_1} + I_{\alpha_2} - I_{\alpha_1} I_{\alpha_2}))^\lambda \right) \left(2! \frac{P_{I_{\alpha_1}} P_{I_{\alpha_2}}}{P_{I_{\alpha_1}} + P_{I_{\alpha_2}}} \right), \\
&\quad \times \left(1 - (1 - (F_{\alpha_1} + F_{\alpha_2} - F_{\alpha_1} F_{\alpha_2}))^\lambda \right) \left(2! \frac{P_{F_{\alpha_1}} P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \Bigg) \\
&= \left(T_{\alpha_1}^\lambda T_{\alpha_2}^\lambda \left(2! \frac{P_{T_{\alpha_1}} P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), \right. \\
&\quad \left(1 - (1 - I_{\alpha_1})^\lambda (1 - I_{\alpha_2})^\lambda \right) \left(2! \frac{P_{I_{\alpha_1}} P_{I_{\alpha_2}}}{P_{I_{\alpha_1}} + P_{I_{\alpha_2}}} \right), \\
&\quad \times \left(1 - (1 - F_{\alpha_1})^\lambda (1 - F_{\alpha_2})^\lambda \right) \left(2! \frac{P_{F_{\alpha_1}} P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \Bigg)
\end{aligned} \tag{3.5}$$

Also we have

$$\begin{aligned}
\alpha_1^\lambda &= \left(T_{\alpha_1}^\lambda (P_{T_{\alpha_1}}), \quad (1 - (1 - I_{\alpha_1})^\lambda) (P_{I_{\alpha_1}}), \right. \\
&\quad \left. (1 - (1 - F_{\alpha_1})^\lambda) (P_{F_{\alpha_1}}) \right) \\
\alpha_2^\lambda &= \left(T_{\alpha_2}^\lambda (P_{T_{\alpha_2}}), \quad (1 - (1 - I_{\alpha_2})^\lambda) (P_{I_{\alpha_2}}), \right. \\
&\quad \left. (1 - (1 - F_{\alpha_2})^\lambda) (P_{F_{\alpha_2}}) \right)
\end{aligned}$$

and

$$\begin{aligned}
\alpha^{\lambda_1} &= \left(T_{\alpha}^{\lambda_1} (P_{T_{\alpha}}), \quad (1 - (1 - I_{\alpha})^{\lambda_1}) (P_{I_{\alpha}}), \right. \\
&\quad \left. (1 - (1 - F_{\alpha})^{\lambda_1}) (P_{F_{\alpha}}) \right); \\
\alpha^{\lambda_2} &= \left(T_{\alpha}^{\lambda_2} (P_{T_{\alpha}}), \quad (1 - (1 - I_{\alpha})^{\lambda_2}) (P_{I_{\alpha}}), \right. \\
&\quad \left. (1 - (1 - F_{\alpha})^{\lambda_2}) (P_{F_{\alpha}}) \right)
\end{aligned}$$

and

$$\begin{aligned}
\alpha^{\lambda_1} \otimes \alpha^{\lambda_2} &= \left(T_{\alpha}^{\lambda_1} T_{\alpha}^{\lambda_2} (P_{T_{\alpha}}), \right. \\
&\quad \left(1 - (1 - I_{\alpha})^{\lambda_1} (1 - I_{\alpha})^{\lambda_2} \right) (P_{I_{\alpha}}), \\
&\quad \left(1 - (1 - F_{\alpha})^{\lambda_1} (1 - F_{\alpha})^{\lambda_2} \right) (P_{F_{\alpha}}) \Bigg) \\
&= \left(T_{\alpha}^{\lambda_1 + \lambda_2} (P_{T_{\alpha}}), \quad (1 - (1 - I_{\alpha})^{\lambda_1 + \lambda_2}) (P_{I_{\alpha}}), \right. \\
&\quad \left. (1 - (1 - F_{\alpha})^{\lambda_1 + \lambda_2}) (P_{F_{\alpha}}) \right) \\
&= \alpha^{\lambda_1 + \lambda_2}
\end{aligned}$$

$$\alpha_1^\lambda \otimes \alpha_2^\lambda = \left(T_{\alpha_1}^\lambda T_{\alpha_2}^\lambda \left(2! \frac{P_{T_{\alpha_1}} P_{T_{\alpha_2}}}{P_{T_{\alpha_1}} + P_{T_{\alpha_2}}} \right), \quad (1 - (1 - I_{\alpha_1})^\lambda (1 - I_{\alpha_2})^\lambda) \left(2! \frac{P_{I_{\alpha_1}} P_{I_{\alpha_2}}}{P_{I_{\alpha_1}} + P_{I_{\alpha_2}}} \right), \right. \\
\left. (1 - (1 - F_{\alpha_1})^\lambda (1 - F_{\alpha_2})^\lambda) \left(2! \frac{P_{F_{\alpha_1}} P_{F_{\alpha_2}}}{P_{F_{\alpha_1}} + P_{F_{\alpha_2}}} \right) \right) \tag{3.6}$$

From the equalities (3.5) and (3.6), we get

$$(\alpha_1 \otimes \alpha_2)^\lambda = \alpha_1^\lambda \otimes \alpha_2^\lambda$$

5. With Definition 3.3 (iii), we have

$$\begin{aligned}
\lambda_1 \alpha &= \left((1 - (1 - T_{\alpha})^{\lambda_1}) (P_{T_{\alpha}}), \quad I_{\alpha}^{\lambda_1} (P_{I_{\alpha}}), \quad F_{\alpha}^{\lambda_1} (P_{F_{\alpha}}) \right); \\
\lambda_2 \alpha &= \left((1 - (1 - T_{\alpha})^{\lambda_2}) (P_{T_{\alpha}}), \quad I_{\alpha}^{\lambda_2} (P_{I_{\alpha}}), \quad F_{\alpha}^{\lambda_2} (P_{F_{\alpha}}) \right)
\end{aligned}$$

and

$$\begin{aligned}
\lambda_1 \alpha \oplus \lambda_2 \alpha &= \left((1 - (1 - T_{\alpha})^{\lambda_1 + \lambda_2}) (P_{T_{\alpha}}), \right. \\
&\quad \left. I_{\alpha}^{\lambda_1 + \lambda_2} (P_{I_{\alpha}}), \quad F_{\alpha}^{\lambda_1 + \lambda_2} (P_{F_{\alpha}}) \right) \\
&= (\lambda_1 + \lambda_2) \alpha
\end{aligned}$$

6. From Definition 3.3 (iv), we have

7. From Definition 3.3 (iv), we have

$$\begin{aligned}
(\alpha^{\lambda_1})^{\lambda_2} &= \left(T_{\alpha}^{\lambda_1} (P_{T_{\alpha}}), \quad (1 - (1 - I_{\alpha})^{\lambda_1}) (P_{I_{\alpha}}), \right. \\
&\quad \left. (1 - (1 - F_{\alpha})^{\lambda_1}) (P_{F_{\alpha}}) \right)^{\lambda_2} \\
&= \left((T_{\alpha}^{\lambda_1})^{\lambda_2} (P_{T_{\alpha}}), \quad (1 - (1 - (1 - I_{\alpha})^{\lambda_1})^{\lambda_2}) (P_{I_{\alpha}}), \right. \\
&\quad \left. (1 - (1 - (1 - I_{\alpha})^{\lambda_1})^{\lambda_2}) (P_{F_{\alpha}}) \right) \\
&= \left(T_{\alpha}^{\lambda_1 \lambda_2} (P_{T_{\alpha}}), \quad (1 - (1 - I_{\alpha})^{\lambda_1 \lambda_2}) (P_{I_{\alpha}}), \right. \\
&\quad \left. (1 - (1 - F_{\alpha})^{\lambda_1 \lambda_2}) (P_{F_{\alpha}}) \right) = \alpha^{\lambda_1 \lambda_2}.
\end{aligned}$$

In the following, score and accuracy functions are defined for PSNN. $\square$

Definition 3.5 Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse and $\alpha = (T(P_T), I(P_I), F(P_F)) \in PSNN(X)$. Then, the score function S of α is defined as follows:

$$S(\alpha) = \frac{TP_T + 1 - IP_I + 1 - FP_F}{3} \quad (3.7)$$

where $S(\alpha) \in [0, 1]$.

Definition 3.6 Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse and $\alpha = (T(P_T), I(P_I), F(P_F)) \in PSNN(X)$. Then, the accuracy function H of α is defined as follows:

$$H(\alpha) = TP_T - FP_F \quad (3.8)$$

where $H(\alpha) \in [-1, 1]$.

A method for comparing $PSNNs$ is given as follows:

Definition 3.7 Let α_1 and α_2 be two $PSNNs$. Then

1. If $S(\alpha_1) > S(\alpha_2)$, then α_1 is greater than α_2 and denoted by $\alpha_1 \succ \alpha_2$.
2. If $S(\alpha_1) = S(\alpha_2)$, then
 - (i) If $H(\alpha_1) > H(\alpha_2)$, then α_1 is greater than α_2 and denoted by $\alpha_1 \succ \alpha_2$.
 - (ii) If $H(\alpha_1) = H(\alpha_2)$, then α_1 is equal to α_2 , denoted by $\alpha_1 = \alpha_2$.

Theorem 3.8 Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse and

$$\begin{aligned} A &= \{ \langle x_k; T_A(x_k)(P_{T_A(x_k)}), I_A(x_k)(P_{I_A(x_k)}), \\ &\quad F_A(x_k)(P_{F_A(x_k)}) \rangle : x_k \in X \} \\ B &= \{ \langle x_k; T_B(x_k)(P_{T_B(x_k)}), I_B(x_k)(P_{I_B(x_k)}), \\ &\quad F_B(x_k)(P_{F_B(x_k)}) \rangle : x_k \in X \} \end{aligned}$$

be two $PSNNs$. The generalized distance measure between A and B is defined below:

$$\begin{aligned} d_G(A, B) &= \left(\frac{1}{3n} \sum_{k=1}^n |T_A P_{T_A}(x_k) - T_B P_{T_B}(x_k)|^\delta + |I_A P_{I_A}(x_k) - I_B P_{I_B}(x_k)|^\delta \right. \\ &\quad \left. + |F_A P_{F_A}(x_k) - F_B P_{F_B}(x_k)|^\delta \right)^{\frac{1}{\delta}} \end{aligned} \quad (3.9)$$

Proof It can be seen that the distance measure satisfies conditions of metric.

Let $C = \{ \langle x_k; T_C(x_k)(P_{T_C(x_k)}), I_C(x_k)(P_{I_C(x_k)}), F_C(x_k)(P_{F_C(x_k)}) \rangle : x_k \in X \}$ be any $PSNNs$. Then,

$$\begin{aligned} \text{(i)} \quad d_G(A, B) &= \left(\frac{1}{3n} \sum_{k=1}^n |T_A(x_k)P_{T_A(x_k)} - T_B(x_k)P_{T_B(x_k)}|^\delta \right. \\ &\quad + |I_A(x_k)P_{I_A(x_k)} - I_B(x_k)P_{I_B(x_k)}|^\delta \\ &\quad \left. + |F_A(x_k)P_{F_A(x_k)} - F_B(x_k)P_{F_B(x_k)}|^\delta \right)^{1/\delta} \geq 0 \end{aligned}$$

(ii) If $A = B$, then $T_A(x_k) = T_B(x_k)$, $P_{T_A(x_k)} = P_{T_B(x_k)}$, $I_A(x_k) = I_B(x_k)$, $P_{I_A(x_k)} = P_{I_B(x_k)}$ and $F_A(x_k) = F_B(x_k)$, $P_{F_A(x_k)} = P_{F_B(x_k)}$ for all $k = 1, 2, \dots, n$. Hence, it is obvious that

$$\begin{aligned} d_G(A, B) &= \left(\frac{1}{3n} \sum_{k=1}^n |T_A(x_k)P_{T_A(x_k)} - T_B(x_k)P_{T_B(x_k)}|^\delta \right. \\ &\quad + |I_A(x_k)P_{I_A(x_k)} - I_B(x_k)P_{I_B(x_k)}|^\delta \\ &\quad \left. + |F_A(x_k)P_{F_A(x_k)} - F_B(x_k)P_{F_B(x_k)}|^\delta \right)^{1/\delta} = 0 \\ \text{(iii)} \quad d_G(A, B) &= \left(\frac{1}{3n} \sum_{k=1}^n |T_A(x_k)P_{T_A(x_k)} - T_B(x_k)P_{T_B(x_k)}|^\delta \right. \\ &\quad + |I_A(x_k)P_{I_A(x_k)} - I_B(x_k)P_{I_B(x_k)}|^\delta \\ &\quad + |F_A(x_k)P_{F_A(x_k)} - F_B(x_k)P_{F_B(x_k)}|^\delta \Big)^{1/\delta} \\ &= \left(\frac{1}{3n} \sum_{k=1}^n |T_B(x_k)P_{T_B(x_k)} - T_A(x_k)P_{T_A(x_k)}|^\delta + |I_B(x_k)P_{I_B(x_k)} - I_A(x_k)P_{I_A(x_k)}|^\delta \right. \\ &\quad \left. + |F_B(x_k)P_{F_B(x_k)} - F_A(x_k)P_{F_A(x_k)}|^\delta \right)^{1/\delta} = d_G(B, A) \\ \text{(iv)} \quad d_G(A, B) &= \left(\frac{1}{3n} \sum_{k=1}^n |T_A(x_k)P_{T_A(x_k)} - T_B(x_k)P_{T_B(x_k)}|^\delta \right. \\ &\quad + |I_A(x_k)P_{I_A(x_k)} - I_B(x_k)P_{I_B(x_k)}|^\delta \\ &\quad + |F_A(x_k)P_{F_A(x_k)} - F_B(x_k)P_{F_B(x_k)}|^\delta \Big)^{1/\delta} \\ &= \left(\frac{1}{3n} \sum_{k=1}^n |T_A(x_k)P_{T_A(x_k)} - T_B(x_k)P_{T_B(x_k)} - T_C(x_k)P_{T_C(x_k)} + T_C(x_k)P_{T_C(x_k)}|^\delta \right. \\ &\quad + |I_A(x_k)P_{I_A(x_k)} - I_B(x_k)P_{I_B(x_k)} - I_C(x_k)P_{I_C(x_k)} + I_C(x_k)P_{I_C(x_k)}|^\delta \\ &\quad + |F_A(x_k)P_{F_A(x_k)} - F_B(x_k)P_{F_B(x_k)} - F_C(x_k)P_{F_C(x_k)} + F_C(x_k)P_{F_C(x_k)}|^\delta \Big)^{1/\delta} \\ &\leq \left(\frac{1}{3n} \sum_{k=1}^n |T_A(x_k)P_{T_A(x_k)} - T_C(x_k)P_{T_C(x_k)}|^\delta + |T_B(x_k)P_{T_B(x_k)} - T_C(x_k)P_{T_C(x_k)}|^\delta \right. \\ &\quad + |I_A(x_k)P_{I_A(x_k)} - I_C(x_k)P_{I_C(x_k)}|^\delta + |I_B(x_k)P_{I_B(x_k)} - I_C(x_k)P_{I_C(x_k)}|^\delta \\ &\quad + |F_A(x_k)P_{F_A(x_k)} - F_C(x_k)P_{F_C(x_k)}|^\delta + |F_B(x_k)P_{F_B(x_k)} - F_C(x_k)P_{F_C(x_k)}|^\delta \Big)^{1/\delta} \\ &\leq \left(\frac{1}{3n} \sum_{k=1}^n |T_A(x_k)P_{T_A(x_k)} - T_C(x_k)P_{T_C(x_k)}|^\delta + |I_A(x_k)P_{I_A(x_k)} - I_C(x_k)P_{I_C(x_k)}|^\delta \right. \\ &\quad + |F_A(x_k)P_{F_A(x_k)} - F_C(x_k)P_{F_C(x_k)}|^\delta \Big)^{1/\delta} \\ &\quad + \left(\frac{1}{3n} \sum_{k=1}^n |T_B(x_k)P_{T_B(x_k)} - T_C(x_k)P_{T_C(x_k)}|^\delta + |I_B(x_k)P_{I_B(x_k)} - I_C(x_k)P_{I_C(x_k)}|^\delta \right. \\ &\quad \left. + |F_B(x_k)P_{F_B(x_k)} - F_C(x_k)P_{F_C(x_k)}|^\delta \right)^{1/\delta} = d_G(A, C) + d_G(B, C) \end{aligned}$$

For $\delta = 1$, we get the probabilistic simplified neutrosophic Hamming distance as

$$\begin{aligned} d_H(A, B) &= \frac{1}{3n} \sum_{k=1}^n (|T_A P_{T_A}(x_k) - T_B P_{T_B}(x_k)| + |I_A P_{I_A}(x_k) \\ &\quad - I_B P_{I_B}(x_k)| + |F_A P_{F_A}(x_k) - F_B P_{F_B}(x_k)|), \end{aligned} \quad (3.10)$$

and for $\delta = 2$, we get the probabilistic simplified neutrosophic Euclidian distance as

$$d_E(A, B) = \left(\frac{1}{3n} \sum_{k=1}^n \left(|T_A(x_k)P_{T_A(x_k)} - T_B(x_k)P_{T_B(x_k)}|^2 + |I_A(x_k)P_{I_A(x_k)} - I_B(x_k)P_{I_B(x_k)}|^2 + |F_A(x_k)P_{F_A(x_k)} - F_B(x_k)P_{F_B(x_k)}|^2 \right) \right)^{\frac{1}{2}} \quad (3.11)$$

Definition 3.9 Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse.

$$A = \{ \langle x_k; T_A(x_k)(P_{T_A(x_k)}), I_A(x_k)(P_{I_A(x_k)}), F_A(x_k)(P_{F_A(x_k)}) \rangle : x_k \in X \}$$

$$B = \{ \langle x_k; T_B(x_k)(P_{T_B(x_k)}), I_B(x_k)(P_{I_B(x_k)}), F_B(x_k)(P_{F_B(x_k)}) \rangle : x_k \in X \}$$

be two PSNSs. The inner product of A and B is defined as

$$AB = \sum_{k=1}^n \left((T_A(x_k) \cdot P_{T_A(x_k)}) \cdot (T_B(x_k)P_{T_B(x_k)}) + (1 - I_A(x_k) \cdot (1 - P_{I_A(x_k)})) \cdot (1 - I_B(x_k) \cdot (1 - P_{I_B(x_k)})) + (1 - F_A(x_k) \cdot (1 - P_{F_A(x_k)})) \cdot (1 - F_B(x_k) \cdot (1 - P_{F_B(x_k)})) \right) \quad (3.12)$$

Definition 3.10 Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse, and

$$A = \{ \langle x_k; T_A(x_k)(P_{T_A(x_k)}), I_A(x_k)(P_{I_A(x_k)}), F_A(x_k)(P_{F_A(x_k)}) \rangle : x_k \in X \} \in PSNS(X).$$

Then,

$$|A| = \sqrt{\sum_{k=1}^n \left((T_A(x_k) \cdot P_{T_A(x_k)})^2 + (1 - I_A(x_k) \cdot (1 - P_{I_A(x_k)}))^2 + (1 - F_A(x_k) \cdot (1 - P_{F_A(x_k)}))^2 \right)} \quad (3.13)$$

is called the module of A .

Now we define a projection measure that takes into account both distance and angle between two sets to determine difference or similarity between PSNSs.

Definition 3.11 Let A and B be two PSNSs. Then, the projection of A on B is defined as

$$Proj_B(A) = \frac{AB}{|B|} \quad (3.14)$$

where

$$|B| = \sqrt{\sum_{k=1}^n \left((T_B(x_k) \cdot P_{T_B(x_k)})^2 + (1 - I_B(x_k) \cdot (1 - P_{I_B(x_k)}))^2 + (1 - F_B(x_k) \cdot (1 - P_{F_B(x_k)}))^2 \right)}$$

Example 3.12 Let

$$A = \{ \langle 0, 5(0, 3), 0, 2(0, 4), 0, 6(0, 7) \rangle \}$$

$$B = \{ \langle 0, 6(0, 4), 0, 4(0, 5), 0, 5(0, 3) \rangle \}$$

be two PSNNs. Then, projection values between A and B are obtained as follows:

$$Proj_A A = 0.517, \quad Proj_B B = 0.520, \quad Proj_A B = 0.429, \quad \text{and} \quad Proj_B A = 0.427$$

The greater the projection value, the closer A and B to each other. A cannot closer to B than itself so the results are reasonable.

Theorem 3.13 Let A, B and C be any three PSNSs. Then, the projection operator satisfies the following properties:

- (i) $0 \leq Proj_B(A) \leq |A| \leq \sqrt{3n}$;
- (ii) If $A \subseteq B$, $Proj_C(A) \leq Proj_C(B)$;
- (iii) $A = B$, $Proj_B(A) = Proj_A(B) = |A| = |B|$.

Now, we further define the normalized projection measure of PSNSs.

Definition 3.14 Let A and B be any two PSNSs. Then, the normalized projection of A on B is defined as

$$NProj_B(A) = \frac{Proj_B(A)}{Proj_B(A) + ||B| - Proj_B(A)|}. \quad (3.15)$$

By Definition 3.14, we can say that the normalized projection operator satisfies the following properties:

- (i) $0 \leq NProj_B(A) \leq 1$;
- (ii) If $A \subseteq B \subseteq C$, $NProj_C(A) \leq NProj_C(B)$;
- (iii) If $A = B$, $NProj_B(A) = 1$.

Definition 3.15 Let $A = \{ \langle x_k; T_A(x_k)(P_{T_A(x_k)}), I_A(x_k)(P_{I_A(x_k)}), F_A(x_k)(P_{F_A(x_k)}) \rangle : x_k \in X \}$ ($k = 1, 2, \dots, n$) be a PSNS. Closeness coefficient of A based on the developed projection measure is defined as

$$RC_A = \frac{Proj_{A^+}(A)}{Proj_{A^+}(A) + Proj_{A^-}(A)} \quad (3.16)$$

where

$$A^+ = \left\{ \langle x_k; \max\{T_A(x_k)\}(\max\{P_{T_A(x_k)}\}), \right. \\ \left. \min\{I_A(x_k)\}(\min\{P_{I_A(x_k)}\}), \right. \\ \left. \min\{F_A(x_k)\}(\min\{P_{F_A(x_k)}\}) \rangle : x_k \in X \right\}$$

and

$$A^- = \left\{ \langle x_k : \min\{T_A(x_k)\}(\min\{P_{T_A(x_k)}\}), \right. \\ \left. \max\{I_A(x_k)\}(\max\{P_{I_A(x_k)}\}), \right. \\ \left. \max\{F_A(x_k)\}(\max\{P_{F_A(x_k)}\}) \rangle : x_k \in X \right\}.$$

Definition 3.16 Let A, B be two PSNSs, then the projection-based difference operator between A and B is defined as

$$Diff(A, B) = RC_A - RC_B \\ = \frac{Proj_{A^+}(A)}{Proj_{A^+}(A) + Proj_{A^-}(A)} \\ - \frac{Proj_{B^+}(B)}{Proj_{B^+}(B) + Proj_{B^-}(B)} \quad (3.17)$$

Theorem 3.17 Let A, B be two PSNSs, then $Diff$ operator satisfies the following properties:

- (i) $-1 \leq Diff(A, B) \leq 1$;
- (ii) If $A = B$, then $Diff(A, B) = 0$;
- (iii) If $A \subseteq B$, then $Diff(A, B) \leq 0$;
- (iv) $Diff(A, B) + Diff(B, A) = 0$.

Proof The proof can be provided obviously. Thus, it is omitted here. $\square$

4 Aggregation operators for PSNSs

It is important to collect data with the least possible loss. Thus, aggregation operators play an important role in decision process. It is clear that in this process, we gather more data. For this reason, it is necessary to define new aggregation operators including probability information below.

Definition 4.1 Let $\alpha_j = (T_j(P_{T_j}), I_j(P_{I_j}), F_j(P_{F_j}))$, $j = (1, 2, \dots, n)$, be a collection of PSNNs. Then, their aggregated value by using (PSNWA) operator is an PSNN and is defined as

$$PSNWA(\alpha_1, \alpha_2, \dots, \alpha_n) = \sum_{j=1}^n w_j \alpha_j \\ = \left(1 - \prod_{j=1}^n (1 - T_j)^{w_j} \left(n! \frac{\prod_{j=1}^n P_{T_j}}{\prod_{k=2}^n \left(\sum_{j=1}^k P_{T_j} \right)} \right), \right. \\ \left. \prod_{j=1}^n I_j^{w_j} \left(n! \frac{\prod_{j=1}^n P_{I_j}}{\prod_{k=2}^n \left(\sum_{j=1}^k P_{I_j} \right)} \right), \right. \\ \left. \prod_{j=1}^n F_j^{w_j} \left(n! \frac{\prod_{j=1}^n P_{F_j}}{\prod_{k=2}^n \left(\sum_{j=1}^k P_{F_j} \right)} \right) \right) \quad (4.1)$$

where w_j is the weight of α_j , $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Remark 1 If $w_j = \frac{1}{n}$ for all $j = 1, 2, \dots, n$, PSNWA operator is reduced to the probabilistic simplified neutrosophic arithmetic (PSNA) operator.

Specially; if $P_{T_j} = P_{I_j} = P_{F_j} = 1$ for $j = 1, 2, \dots, n$, PSNWA operator is reduced to the simplified neutrosophic weighted arithmetic (SNWA) operator given by Peng et al. (2016).

Definition 4.2 Let $\alpha_j = (T_j(P_{T_j}), I_j(P_{I_j}), F_j(P_{F_j}))$, $j = (1, 2, \dots, n)$, be a collection of PSNNs. Then, their aggregated value by using (PSNWG) operator is an PSNN and defined by

$$PSNWG(\alpha_1, \alpha_2, \dots, \alpha_n) \\ = \left(\prod_{j=1}^n T_j^{w_j} \left(n! \frac{\prod_{j=1}^n P_{T_j}}{\prod_{k=2}^n \left(\sum_{j=1}^k P_{T_j} \right)} \right), \right. \\ \left. 1 - \prod_{j=1}^n (1 - I_j)^{w_j} \left(n! \frac{\prod_{j=1}^n P_{I_j}}{\prod_{k=2}^n \left(\sum_{j=1}^k P_{I_j} \right)} \right), \right. \\ \left. 1 - \prod_{j=1}^n (1 - F_j)^{w_j} \left(n! \frac{\prod_{j=1}^n P_{F_j}}{\prod_{k=2}^n \left(\sum_{j=1}^k P_{F_j} \right)} \right) \right) \quad (4.2)$$

where w_j is the weight of α_j , $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Remark 2 If $w_j = \frac{1}{n}$ for all $j = 1, 2, \dots, n$, PSNWG operator is reduced to the probabilistic simplified neutrosophic geometric (PSNG) operator.

Specially, if $P_{T_j} = P_{I_j} = P_{F_j} = 1$ for $j = 1, 2, \dots, n$, PSNWG operator is reduced to the simplified neutrosophic weighted geometric (SNWG) operator given by Peng et al. (2016).

5 Extended PROMETHEE method based on PSNS information

In multi-criteria decision problems, the importance of weight of each decision maker cannot be ignored. It has a direct effect on the results. So it is important to determine weights of decision makers more objectively. In this section, we will present a method using by preference function to determine weights of each decision maker. Let $A = \{A_1, A_2, \dots, A_n\}$ be the set of alternatives, $C = \{C_1, C_2, \dots, C_m\}$ be the set of criteria, $D = \{D_1, D_2, \dots, D_t\}$ be the set of decision makers and $R = (r_{sj}^p)$ ($s = 1, 2, \dots, n$; $j = 1, 2, \dots, m$; $p = 1, 2, \dots, t$) be the decision matrix of alternatives according to the criteria given by decision maker.

5.1 Determination of decision maker's weights

Now, we presented an approach to determine the weights of decision makers. Its steps are as follows:

Step 1 Constitute average decision matrix which consists of PSNNs and its complement matrix by using the probabilistic simplified neutrosophic arithmetic average (PSNA) operator as follows:

$$\begin{aligned} D^+ &= (r_{sj}^+) = PSNA(r_{sj}^1, r_{sj}^2, \dots, r_{sj}^t) \quad \text{and} \\ D^- &= (r_{sj}^-) = (PSNA(r_{sj}^1, r_{sj}^2, \dots, r_{sj}^t))^c \end{aligned} \quad (5.1)$$

Step 2 Define positive and negative preference value of each decision maker's matrix according to average matrices D^+ and D^- , respectively, as

$$\varepsilon_p^+ = P(D^p, D^+) \quad \text{and} \quad \varepsilon_p^- = P(D^-, D^p) \quad (p = 1, 2, \dots, t) \quad (5.2)$$

where D^p is the matrix of the p th decision maker. $P(D^p, D^+)$ and $P(D^-, D^p)$ denote the information of preference of D^p to D^+ and preference of D^- to D^p , respectively.

We use linear preference function in this calculation defined as follows:

$$P(D^p, D^+) = \begin{cases} 0, & d(D^p, D^+) \leq e \\ \frac{d(D^p, D^+) - e}{f - e}, & e < d(D^p, D^+) \leq f \\ 1, & d(D^p, D^+) \geq f \end{cases} \quad (5.3)$$

$$P(D^-, D^p) = \begin{cases} 0, & d(D^-, D^p) \leq e \\ \frac{d(D^-, D^p) - e}{f - e}, & e < d(D^-, D^p) \leq f \\ 1, & d(D^-, D^p) \geq f \end{cases} \quad (5.4)$$

where e and f are parameters of indifference and strict preference values, respectively, and

$$\begin{aligned} d(D^p, D^+) &= \frac{1}{n} \frac{1}{m} \sum_{j=1}^m \sum_{s=1}^n d_E(r_{sj}^p, r_{sj}^+) \\ d(D^-, D^p) &= \frac{1}{n} \frac{1}{m} \sum_{j=1}^m \sum_{s=1}^n d_E(r_{sj}^-, r_{sj}^p) \end{aligned} \quad (5.5)$$

where d_E is the metric function defined in Eq. (3.11)

Step 3 Calculate the net preference value of decision maker D^p as

$$\varepsilon_p^{\text{net}} = \chi \varepsilon_p^+ + (1 - \chi) \varepsilon_p^-, \quad (p = 1, 2, \dots, t) \quad (5.6)$$

where χ is a risk index and

$$\varepsilon_p^+ = 1 - \frac{P(D^p, D^+)}{\sum_{p=1}^t P(D^p, D^+)}, \quad \varepsilon_p^- = \frac{P(D^p, D^-)}{\sum_{p=1}^t P(D^p, D^-)} \quad (5.7)$$

Step 4 Obtain the weight of the decision maker D^p as

$$\gamma_p = \frac{\varepsilon_p^{\text{net}}}{\sum_{p=1}^t \varepsilon_p^{\text{net}}}, \quad p = 1, 2, \dots, t \quad (5.8)$$

5.2 Determine weight of criteria with fuzzy measure

Definition 5.1 [Fuzzy measure (Wang and Klir 1992; Pap 1995; Sugeno 1974)] A fuzzy measure on the universal set X is a set function $\mu: P(X) \rightarrow [0, 1]$ satisfying the following properties:

1. $\mu(\emptyset) = 0$ and $\mu(X) = 1$,
2. If $A, B \in P(X)$ and $A \subseteq B$, then $\mu(A) \leq \mu(B)$.

In decision making process, there may be an interaction between criteria and decision maker that can take into consideration any combinations of criteria besides the weight of each element. Decision maker can give a priority to any combination of criteria elements.

With the fuzzy measure function, weight of elements in C and their combinations can be expressed.

According to the Sugeno's λ -fuzzy measure (Sugeno 1974), weight of combination $C_y, C_z \in C$ ($y = 1, 2, \dots, m$, $z = 1, 2, \dots, m$) is calculated with the following equation:

$$\mu(C_y \cup C_z) = \mu(C_y) + \mu(C_z) + \lambda \mu(C_y) \mu(C_z), \quad (5.9)$$

$$A \cap B = \emptyset$$

If $\lambda = 0$, then μ is additive measure, i.e.,

$$\mu(C_y \cup C_z) = \mu(C_y) + \mu(C_z) \quad y = 1, 2, \dots, m; \quad z = 1, 2, \dots, m \quad (5.10)$$

If $\lambda > 0$, then μ is super-additive measure, i.e., when criteria C_y and C_z are considered together, the weight measure increases so the criteria have a positive effect on each other.

If $\lambda < 0$, then μ is sub-additive measure, i.e.,

$$\mu(C_y \cup C_z) < \mu(C_y) + \mu(C_z) \quad (5.11)$$

We can say they have negative effect on each other.

The parameter λ can be determined with the equation given by Sugeno (1974) as

$$C = \bigcup_{y=1}^m C_y$$

and

$$\mu(C) = \mu\left(\bigcup_{y=1}^m C_y\right) = \begin{cases} \frac{1}{\lambda} \left(\prod_{y=1}^m (1 + \lambda \mu(C_y)) - 1 \right), & \lambda \neq 0 \\ \sum_{y=1}^m \mu(C_y), & \lambda = 0 \end{cases} \quad (5.12)$$

Let $P(C)$ be the power set of C . For any subset $C_y \in P(C)$ ($y = 1, 2, \dots, m$), the fuzzy measure is $\mu(C_y)$ defined as

$$\mu(C_y) = \begin{cases} \frac{1}{\lambda} \left(\prod_{x_y \in C_y} (1 + \lambda \mu(x_y)) - 1 \right), & \lambda \neq 0 \\ \sum_{x_y \in C_y} \mu(x_y), & \lambda = 0 \end{cases} \quad (5.13)$$

Since $\mu(C) = 1$, we obtain the value of λ in the following equation:

$$\lambda + 1 = \prod_{y=1}^m (1 + \lambda \mu(C_y)) \quad (5.14)$$

With the solution equation Eq. (5.13), λ is determined, and fuzzy measures of combinations of criteria can be calculated using by Eq. (5.11).

Now we calculate the relative weights of criteria for each alternative. In decision matrix, evaluation values are rearranged in descending order for each alternative according to the criteria to consider relationship between criteria, and we suppose that $r_{s\sigma(1)} \succ r_{s\sigma(2)} \succ \dots \succ r_{s\sigma(n)}$ ($s = 1, 2, \dots, n$) for sth alternative and $C_{s\sigma(j)}$ is the criterion

related to the $r_{s\sigma(j)}$. Finally, using λ -fuzzy measure, weights of criteria over each alternative are calculated as

$$w_{s\sigma(j)} = \mu(A_{s\sigma(j)}) - \mu(A_{s\sigma(j-1)}) \quad (5.15)$$

where $A_{s\sigma(j)} = \{C_{\sigma(t)} : t \leq j, j \geq 1\}$ ($j = 1, 2, \dots, m$), $A_{k\sigma(0)} = \emptyset$.

Here, $w_{s\sigma(j)}$ is the weight of criterion $C_{s\sigma(j)}$.

5.3 Classical PROMETHEE method

Preference ranking organization method for enrichment evaluation (PROMETHEE) was first defined by Mareschal and Vincke (1984). It makes possible binary comparison of alternatives according to the given criteria. The steps to be followed in this method are as follows:

Step 1 Constitute the evaluation values matrix of alternatives A_s ($s = 1, 2, \dots, n$) according to the criteria C_j . Then, for each criterion the difference between alternatives is calculated as

$$d_j(A_s, A_v) = p_j(A_s) - p_j(A_v) \quad (s = 1, 2, \dots, n; \quad v = 1, 2, \dots, n), \quad (j = 1, 2, \dots, m) \quad (5.16)$$

where the difference between alternatives A_s and A_v over criteria C_j is denoted by $d_j(A_s, A_v)$.

Step 2 Determine superiorities between alternatives for each criterion as

$$P_j(A_s, A_v) = f_j(d_j(A_s, A_v)) = \begin{cases} 0, & d_j(A_s, A_v) \leq e \\ \frac{d_j(A_s, A_v) - e}{f - e}, & e \leq d_j(A_s, A_v) \leq f \\ 1, & d_j(A_s, A_v) \geq f \end{cases} \quad (5.17)$$

where f_j is a preference function determined by decision makers. We use the linear preference function which assigns the difference between alternatives to the values between 0 and 1.

Step 3 Calculate the preference index of each alternative using the weights of criteria as

$$\pi(A_s, A_v) = \sum_{j=1}^n w_j P_j(A_s, A_v) \quad (5.18)$$

Step 3 Calculate each alternative positive flow φ^+ and negative flow φ^- , respectively, as

$$\varphi^+(A_s) = \frac{1}{n-1} \sum_{\substack{v=1 \\ v \neq s}}^n \pi(A_s, A_v) \quad (5.19)$$

$$\varphi^-(A_s) = \frac{1}{n-1} \sum_{\substack{v=1 \\ v \neq s}}^n \pi(A_v, A_s) \quad (5.20)$$

Thus, partial ranking between alternatives is obtained as follows:

$$r_{ij} = PSNWA(r_{ij}^1, r_{ij}^2, \dots, r_{ij}^t) = \left(1 - \prod_{p=1}^t (1 - T_{ij}^p)^{\gamma_p} \left(t! \frac{\prod_{p=1}^t P_{T_{ij}}^p}{\prod_{l=2}^t (\sum_{p=1}^l P_{T_{ij}}^p)} \right), \prod_{p=1}^t (I_{ij}^p)^{\gamma_p} \left(t! \frac{\prod_{p=1}^t P_{I_{ij}}^p}{\prod_{l=2}^t (\sum_{p=1}^l P_{I_{ij}}^p)} \right), \prod_{p=1}^t (F_{ij}^p)^{\gamma_p} \left(t! \frac{\prod_{p=1}^t P_{F_{ij}}^p}{\prod_{l=2}^t (\sum_{p=1}^l P_{F_{ij}}^p)} \right) \right) \quad (5.22)$$

1. If $\varphi^+(A_s) > \varphi^+(A_v)$ and $\varphi^-(A_s) < \varphi^-(A_v)$, then A_s is preferable to A_v
2. If $\varphi^+(A_s) = \varphi^+(A_v)$ and $\varphi^-(A_s) = \varphi^-(A_v)$, then there is no difference between A_s and A_v
3. If $\varphi^+(A_s) > \varphi^+(A_v)$ and $\varphi^-(A_s) > \varphi^-(A_v)$ or $\varphi^+(A_s) < \varphi^+(A_v)$ and $\varphi^-(A_s) < \varphi^-(A_v)$, the comparison cannot be possible.

Step 4 Net flow is calculated as

$$\varphi(A_s) = \varphi^+(A_s) - \varphi^-(A_s) \quad (5.21)$$

In this way, total order of alternatives is obtained.

5.4 Multi-criteria group decision making method with PSNNs

To choose the best alternative with PROMETHEE in probabilistic simplified neutrosophic environment, we will follow the steps below:

Step 1 First, we construct the $R^p = (r_{ij}^p)$ decision matrix.

r_{ij}^p be the assessment of the alternative A_i with respect to the criteria C_j of the decision maker D_p . Here, for all $i = 1, 2, \dots, n$, $j = 1, 2, \dots, m$, $p = 1, 2, \dots, t$, $r_{ij}^p = (T_{ij}^p(P_{T_{ij}}^p), I_{ij}^p(P_{I_{ij}}^p), F_{ij}^p(P_{F_{ij}}^p))$ and $0 \leq T_{ij}^p + I_{ij}^p + F_{ij}^p \leq 3$ and $0 < P_{T_{ij}}^p, P_{I_{ij}}^p, P_{F_{ij}}^p \leq 1$.

Step 2 Calculate the weights of all decision makers γ_p ($p = 1, 2, \dots, t$) by using the method given in Sect. 5.1

Step 3 Constitute the average group decision matrix $R = (r_{ij}) = (T_{ij}(P_{T_{ij}}), I_{ij}(P_{I_{ij}}), F_{ij}(P_{F_{ij}}))$ using the aggregation operator which is defined in Sect. 4 as

Step 4 Determine the weight vector of criteria $w_s = (w_{s1}, w_{s2}, \dots, w_{sm})^T$ ($s = 1, 2, \dots, n$) according to each alternative by using λ -fuzzy measure as mentioned in Sect. 5.2.

Step 5 Compare pairwise the alternatives over each criterion with preference function based on Diff operator as follows:

$$P_j(A_s, A_v) = f_j(\text{Diff}_j(A_s, A_v)) \quad s = 1, 2, \dots, n, \quad v = 1, 2, \dots, n, \quad j = 1, 2, \dots, m \quad (5.23)$$

where

$$\text{Diff}_j(A_s, A_v) = RC_{A_{sj}} - RC_{A_{vj}}, \quad (5.24)$$

$$RC_{A_{sj}} = \frac{\text{Proj}(A_{sj}, A_j^+)}{\text{Proj}(A_{sj}, A_j^+) + \text{Proj}(A_{sj}, A_j^-)}, \quad (5.25)$$

$$RC_{A_{vj}} = \frac{\text{Proj}(A_{vj}, A_j^+)}{\text{Proj}(A_{vj}, A_j^+) + \text{Proj}(A_{vj}, A_j^-)} \quad (5.26)$$

and

$$A^+ = (\max\{T_{ij}\}(\max\{P_{T_{ij}}\}), \min\{I_{ij}\}(\min\{P_{I_{ij}}\}), \min\{F_{ij}\}(\min\{P_{F_{ij}}\})),$$

$$A^- = (\min\{T_{ij}\}(\min\{P_{T_{ij}}\}), \max\{I_{ij}\}(\max\{P_{I_{ij}}\}), \max\{F_{ij}\}(\max\{P_{F_{ij}}\}))$$

We use preference function type of criterion with linear preference and indifference area function defined as

Table 1 Decision matrix given by D_1

	C_1	C_2	C_3
A_1	(0.6,(0.8),0.3(0.9),0.2(0.9))	(0.8(0.8),0.1(0.9),0.1(0.7))	(0.8(0.8),0.1(1),0.2(0.7))
A_2	(0.8(1),0.3(0.8),0.5(0.7))	(0.7(0.9),0.2(1),0.4(0.8))	(0.8(0.9),0.2(0.7),0.3(1))
A_3	(0.8(0.9),0.2(0.7),0.3(1))	(0.6(1),0.1(0.9),0.2(1))	(0.5(0.9),0.3(0.9),0.5(0.8))
A_4	(0.7(1),0.2(1),0.2(0.7))	(0.8(0.8),0.2(0.9),0.2(0.9))	(0.7(1),0.4(1),0.1(0.9))
A_5	(0.8(0.9),0.1(0.9),0.3(0.8))	(0.8(1),0.3(0.8),0.2(1))	(0.8(1),0.4(0.7),0.2(0.9))
	C_4	C_5	
A_1	(0.6(0.9),0.2(0.9),0.3(0.7))	(0.7(1),0.4(1),0.3(1))	
A_2	(0.7(0.8),0.2(1),0.1(0.8))	(0.8(0.9),0.1(1),0.1(0.9))	
A_3	(0.7(0.8),0.3(0.8),0.3(1))	(0.7(0.8),0.2(0.8),0.2(0.9))	
A_4	(0.8(0.9),0.1(1),0.2(0.8))	(0.6(0.9),0.3(1),0.3(1))	
A_5	(0.8(1),0.1(0.7),0.2(0.9))	(0.5(1),0.1(0.8),0.2(0.8))	

Table 2 Decision matrix given by D_2

	C_1	C_2	C_3
A_1	(0.3(0.7),0.2(0.9),0.3(0.9))	(0.6(0.9),0.2(0.7),0.1(0.8))	(0.4(1),0.1(1),0.2(0.9))
A_2	(0.6(0.6),0.8(0.7),0.4(1))	(0.7(0.7),0.2(0.9),0.1(1))	(0.6(0.8),0.2(0.5),0.1(0.8))
A_3	(0.8(1),0.2(0.9),0.1(0.8))	(0.5(0.8),0.3(0.7),0.2(0.8))	(0.3(1),0.4(0.7),0.5(1))
A_4	(0.7(0.8),0.2(1),0.1(0.8))	(0.6(0.6),0.1(1),0.3(0.9))	(0.5(0.8),0.2(1),0.1(0.8))
A_5	(0.8(0.9),0.1(0.8),0.6(0.7))	(0.8(1),0.4(0.8),0.2(1))	(0.4(0.9),0.2(0.9),0.1(1))
	C_4	C_5	
A_1	(0.8(0.8),0.1(1),0.2(0.8))	(0.5(0.8),0.4(1),0.2(0.9))	
A_2	(0.8(0.7),0.2(0.9),0.2(0.8))	(0.8(0.9),0.2(0.9),0.1(0.7))	
A_3	(0.7(0.8),0.3(0.9),0.2(1))	(0.7(1),0.2(0.8),0.6(1))	
A_4	(0.6(1),0.1(0.8),0.3(0.5))	(0.4(0.6),0.3(0.9),0.3(0.9))	
A_5	(0.4(0.9),0.3(1),0.3(1))	(0.6(0.8),0.2(1),0.2(1))	

Table 3 Decision matrix given by D_3

	C_1	C_2	C_3
A_1	(0.7(1),0.2(0.9),0.1(0.8))	(0.6(0.6),0.2(1),0.1(1))	(0.5(0.8),0.2(1),0.1(0.9))
A_2	(0.8(0.9),0.1(0.8),0.4(1))	(0.8(0.9),0.2(0.9),0.1(0.9))	(0.8(1),0.2(0.9),0.6(1))
A_3	(0.7(1),0.2(0.8),0.2(0.9))	(0.7(0.7),0.4(1),0.2(0.7))	(0.5(0.9),0.7(0.8),0.5(1))
A_4	(0.5(0.9),0.6(0.7),0.2(0.8))	(0.5(1),0.5(0.8),0.4(0.9))	(0.8(0.9),0.3(1),0.7(0.8))
A_5	(0.6(0.9),0.6(1),0.3(0.9))	(0.7(0.8),0.6(0.7),0.3(0.8))	(0.6(1),0.2(0.9),0.6(0.8))
	C_4	C_5	
A_1	(0.4(1),0.2(1),0.1(1))	(0.8(0.9),0.5(1),0.2(1))	
A_2	(0.5(0.7),0.2(0.9),0.1(0.8))	(0.8(1),0.1(0.7),0.2(0.7))	
A_3	(0.6(1),0.2(1),0.6(1))	(0.7(0.9),0.1(0.8),0.4(0.8))	
A_4	(0.8(0.9),0.1(0.7),0.6(0.9))	(0.6(0.8),0.2(1),0.2(0.9))	
A_5	(0.5(0.9),0.4(0.8),0.4(0.8))	(0.5(0.8),0.2(0.8),0.6(1))	

Table 4 Generalized group decision matrix

	C_1	C_2
A_1	(0.558(0.896),0.234(0.900),0.187(0.831))	(0.695(0.663),0.153(0.909),0.100(0.896))
A_2	(0.749(0.810),0.302(0.779),0.436(0.915))	(0.733(0.851),0.200(0.914),0.172(0.889))
A_3	(0.776(0.980),0.200(0.788),0.187(0.889))	(0.604(0.747),0.212(0.909),0.200(0.747))
A_4	(0.653(0.889),0.273(0.778),0.160(0.779))	(0.675(0.857),0.207(0.842),0.278(0.900))
A_5	(0.756(0.900),0.167(0.941),0.376(0.840))	(0.776(0.857),0.401(0.730),0.224(0.857))
	C_3	C_4
A_1	(0.629(0.821),0.122(1),0.164(0.851))	(0.642(0.941),0.160(0.980),0.192(0.896))
A_2	(0.749(0.941),0.200(0.851),0.256(0.952))	(0.696(0.713),0.200(0.914),0.125(0.800))
A_3	(0.442(0.914),0.419(0.788),0.500(0.952))	(0.674(0.923),0.267(0.941),0.320(1))
A_4	(0.684(0.889),0.294(1),0.174(0.813))	(0.749(0.914),0.1(0.747),0.312(0.755))
A_5	(0.652(0.980),0.262(0.851),0.218(0.842))	(0.629(0.914),0.212(0.791),0.278(0.842))
	C_5	
A_1	(0.684(0.889),0.426(1),0.234(0.980))	
A_2	(0.800(0.964),0.125(0.765),0.122(0.719))	
A_3	(0.700(0.889),0.164(0.800),0.348(0.842))	
A_4	(0.544(0.751),0.267(0.980),0.267(0.914))	
A_5	(0.535(0.821),0.153(0.821),0.273(0.952))	

$$P_j(A_s, A_v) = \begin{cases} 0, & |Diff_j(A_s, A_v)| \leq e \\ \frac{|Diff_j(A_s, A_v)| - e}{f - e}, & e < |Diff_j(A_s, A_v)| \leq f \\ 1, & |Diff_j(A_s, A_v)| \geq f \end{cases} \quad (5.27)$$

Here, e and f are threshold values given by decision maker.

Step 6 Calculate preference indexes of alternatives by considering all criteria as:

$$\pi(A_s, A_v) = \sum_{j=1}^m w_{svj} P_j(A_s, A_v), \quad s = 1, 2, \dots, n; \\ v = 1, 2, \dots, n \quad (5.28)$$

where $w_{svj} = w_{sj}^\theta w_{vj}^{1-\theta}$, $\theta \in [0, 1]$, and w_{sj} and w_{vj} are the weights of criterion C_j according to the alternatives A_s and A_v , respectively.

Step 7 Express positive and negative outranking flow for each alternative expressed as:

$$\varphi^+(A_s) = \frac{1}{n-1} \sum_{v=1, v \neq s}^n \pi(A_s, A_v) \quad (5.29)$$

and

$$\varphi^-(A_s) = \frac{1}{n-1} \sum_{v=1, v \neq s}^n \pi(A_v, A_s) \quad (5.30)$$

By the above equation, partial ranking (PROMETHEE I) between two alternatives can be obtained.

Step 8 For complete ranking (PROMETHEE II), net outranking values are calculated as

$$\varphi(A_s) = \varphi^+(A_s) - \varphi^-(A_s) \quad (5.31)$$

Finally, using Eq. (5.31), we can obtain complete ranking of each alternative.

6 An application

Energy demand is increasingly becoming the world's most important problem. Today, when we use fossil fuels in general, there is a search for alternative sources to reduce air, water and environmental pollution. In this problem, we have five alternatives: geothermal (A_1), wind energy (A_2), hydropower (A_3), solar power (A_4) and biomass (A_5). We will consider the alternatives with respect to the criteria: quality of energy source (C_1), environmental (C_2), economic (C_3), technological (C_4) and sociopolitical (C_5). The evaluation values given by decision makers $\{D_1, D_2, D_3\}$ are given in Tables 1, 2 and 3. Fuzzy measures of the criteria are determined, arbitrarily. Assume that $\mu(\emptyset) = 0$, $\mu(C_1) = 0.35$, $\mu(C_2) = 0.3$, $\mu(C_3) = 0.25$, $\mu(C_4) = 0.2$ and $\mu(C_5) = 0.4$, and the decision matrices provided by DMs are presented in Tables 1, 2 and 3.

We use the method mentioned in Sect. 5 to choose the best alternative.

Table 5 λ -fuzzy measure

Criteria		Criteria		Criteria		Criteria	
$\{\emptyset\}$	0	$\{C_1, C_4\}$	0.617	$\{C_1, C_2, C_3\}$	0.732	$\{C_2, C_4, C_5\}$	0.79
$\{C_1\}$	0.35	$\{C_1, C_5\}$	0.69	$\{C_1, C_2, C_4\}$	0.79	$\{C_3, C_4, C_5\}$	0.732
$\{C_2\}$	0.3	$\{C_2, C_3\}$	0.459	$\{C_1, C_2, C_5\}$	0.848	$\{C_1, C_2, C_3, C_4\}$	0.881
$\{C_3\}$	0.25	$\{C_2, C_4\}$	0.538	$\{C_1, C_3, C_4\}$	0.732	$\{C_1, C_2, C_3, C_5\}$	0.931
$\{C_4\}$	0.2	$\{C_2, C_5\}$	0.617	$\{C_1, C_3, C_5\}$	0.795	$\{C_1, C_2, C_4, C_5\}$	0.973
$\{C_5\}$	0.4	$\{C_3, C_4\}$	0.459	$\{C_1, C_4, C_5\}$	0.848	$\{C_1, C_3, C_4, C_5\}$	0.931
$\{C_1, C_2\}$	0.617	$\{C_3, C_5\}$	0.545	$\{C_2, C_3, C_4\}$	0.664	$\{C_2, C_3, C_4, C_5\}$	0.881
$\{C_1, C_3\}$	0.545	$\{C_4, C_5\}$	0.617	$\{C_2, C_3, C_5\}$	0.732	$\{C_1, C_2, C_3, C_4, C_5\}$	1

Table 6 The weights of criteria

	w_1	w_2	w_3	w_4	w_5
A_1	0.199	0.209	0.250	0.069	0.273
A_2	0.273	0.069	0.259	0.200	0.199
A_3	0.317	0.300	0.115	0.068	0.199
A_4	0.119	0.149	0.259	0.200	0.273
A_5	0.231	0.125	0.027	0.217	0.400

Step 1 Firstly, using the method given in Sect. 5.1, $\gamma = (\gamma_p)$ ($p = 1, 2, 3$) weights of each decision maker are calculated as 0.390, 0.325 and 0.285, respectively, ($e = 0.05, f = 0.4$).

Step 2 The generalized group decision by using the PSNWA operator is constituted as Table 4.

Step 3 The value λ is calculated as -0.688 by using the equation given in Eq. (5.13), and λ -fuzzy measure of criteria and their combination are denoted by Eq. (5.11) in Table 5.

Step 4. Determine the weights of criteria for alternatives A_s ($s = 1, 2, 3, 4, 5$).

First, we arrange the evaluation values in descending order for each alternative. Here, we compute the weights of criteria for alternative A_1 . The evaluation values (r_{1j}) of alternative A_1 according to the criteria C_j ($j = 1, 2, 3, 4, 5$) are rearranged as $S(r_{13}) > S(r_{12}) > S(r_{15}) > S(r_{11}) > S(r_{14})$. Thus, the criteria are ordered

as $\{C_3, C_2, C_5, C_1, C_4\}$. Using Eq. (5.14), weights of criteria according to the alternative A_1 are calculated as $w_3 = \mu(C_3) - \mu(\emptyset) = 0.25$, $w_2 = \mu(C_2, C_3) - \mu(C_3) = 0.209$, $w_5 = \mu(C_2, C_3, C_5) - \mu(C_2, C_3) = 0.273$, $w_1 = \mu(C_1, C_2, C_3, C_5) - \mu(C_2, C_3, C_5) = 0.199$ and $w_4 = \mu(C_1, C_2, C_3, C_4, C_5) - \mu(C_1, C_2, C_3, C_5) = 0.069$.

Similarly, weights of criteria according to the other alternatives can be obtained. In Table 6, weights of criteria over each alternative are denoted.

Step 5 The pairwise comparisons of alternatives for each criteria with the linear preference function are calculated as

$$P_j(A_s, A_v) = f_j(\text{Diff}_j(A_s, A_v)) \quad j = 1, 2, 3, 4, 5; \quad (5.32)$$

$$s = 1, 2, 3, 4, 5, \quad v = 1, 2, 3, 4, 5$$

Here, threshold values e and f are taken as 0.05 and 0.2, respectively. Related to this step, the closeness

Table 7 Closeness coefficient of alternatives

π	A_1	A_1	A_1	A_1	A_1
A_1	—	0.305	0.251	0.205	0.330
A_2	0.305	—	0.340	0.350	0.301
A_3	0.227	0.323	—	0.421	0.324
A_4	0.290	0.350	0.421	—	0.040
A_5	0.330	0.301	0.324	0.04	—

Table 8 Preference index matrix of alternatives

π	A_1	A_2	A_3	A_4	A_5
A_1	–	0.305	0.251	0.205	0.330
A_2	0.305	–	0.340	0.350	0.301
A_3	0.227	0.323	–	0.421	0.324
A_4	0.290	0.350	0.421	–	0.040
A_5	0.330	0.301	0.324	0.04	–

Table 9 Outranking values of alternatives

	$\varphi^+(A_s)$	$\varphi^-(A_s)$	$\varphi^{\text{net}}(A_s)$
A_1	0.273	0.288	– 0.015
A_2	0.248	0.320	– 0.072
A_3	0.267	0.334	– 0.067
A_4	0.203	0.254	– 0.051
A_5	0.166	0.249	– 0.082

coefficient of each alternative according to each criterion is given in Table 7.

Step 6 The preference index matrix of alternatives is demonstrated in Table 8. ($\theta = 0.5$).

Step 7 Then, positive and negative outranking flow of each alternative is calculated in Table 9.

In Table 9, since we could not get a ranking according to the positive and negative outranking flow of each alternative (PROMETHEE I), we need the net flow of each alternative (PROMETHEE II).

Finally, we have the ranking as $A_1 > A_4 > A_3 > A_2 > A_5$ according to the net outranking values so we can say that the best option is A_1 .

7 A comparison with other studies existing in literature and discussion

Recently, neutrosophic set theory has been applied to many areas such as medicine, economics, industry and engineering. However, this theory is insufficient to model inadequacy, deficiency and uncertainty in the universe we live in. In this paper, a new set theory called simplified neutrosophic set theory is proposed by combining neutrosophic set theory with probability theory, which considers the risky situation of decision maker.

To test how practical and reliable the proposed method, we have applied various methods existing in the literature on the energy source problem. Later, the ranking results are obtained for each method and the reasons of differences

Table 10 Results for different methods

MCDM methods	Preference order	Best option	Worst option
Şahin and Liu (2017)	$A_1 > A_4 > A_2 > A_5 > A_3$	A_1	A_3
Ye (2014c)	$A_2 > A_1 > A_4 > A_5 > A_3$	A_2	A_3
Şahin and Küçük (2015)	$A_2 > A_4 > A_3 > A_1 > A_5$	A_2	A_5
Peng et al. (2016)			
SNWA	$A_2 > A_1 > A_4 > A_5 > A_3$	A_2	A_3
SNWG	$A_2 > A_1 > A_4 > A_5 > A_3$	A_2	A_3
Küçük and Şahin (2018)	$A_2 > A_1 > A_4 > A_5 > A_3$	A_2	A_3
Şahin and Liu (2016)	$A_1 > A_4 > A_2 > A_5 > A_3$	A_1	A_3
The extended PROMETHEE without possibility values	$A_1 > A_4 > A_2 > A_5 > A_3$	A_1	A_3

between the proposed method and others are explained. The results are shown in Table 10.

We apply the proposed process with the preferences without possibility values in which we get the classical simplified neutrosophic information. Thus, our result is same with the ones of Şahin and Liu (2016, 2017b), while it is partially consistent with the results of the other existing methods. The reasons for the difference are that both PIS (positive ideal solution) and NIS (negative ideal solution) are included to the decision process in our method and we consider individual weights of criteria for each different alternative. More clearly, we develop a method to determine weights of criteria which satisfies to express weights as relative with respect to each alternative. Also, while each weight of criteria is same according to each alternative in classical PROMETHEE, we assign different weights of criteria to each alternative, i.e., the importance degree of the criteria is changed with respect to each alternative. This situation meets the uncertainty in daily life better. This is another reason for the difference.

Obviously, to determine weights of decision makers more objectively, we present a new method based on the preference function. Instead of determining weights as in the other methods, we evaluate the dominance of each decision maker to the process. More precisely, we detect the positive and negative dominance of decision makers and determine the weights accordingly. Generally, PROMETHEE method developed in this study has some advantages over other existing studies. These are summarized below:

1. Performance values provided by decision makers are based on probabilistic simplified neutrosophic information unlike others.
2. The weights of decision makers are determined by using the components of PROMETHEE (preference functions).
3. In order to rank the alternatives, the weight vector of the criteria is determined according to each alternative with the help of fuzzy measure which satisfies to consider the relationship between the relevant criteria.
4. Unlike the classical PROMETHEE method, preference indexes of alternatives are calculated by taking the weight of combined criterion determined by combinative weight determination method.
5. *Diff* function which is based on the projection operator is used instead of distance measure to determine the preferences between alternatives over each criterion.
6. By including both PIS and NIS to the decision process, more precise results are obtained.

8 Conclusions

Through the paper, firstly we defined a new set theory called a probabilistic simplified neutrosophic set (PSNS). There is always an uncertainty where there is a human factory, so in addition to classical simplified neutrosophic sets there may be a need to possibility. For this necessary, we combined the concepts of neutrosophic and probability to get more comprehensive information. Also some of algebraic properties of this set are discussed. Later, some aggregation operators which play an important role in decision process are defined based on the new set theory, and we presented some of their properties. Subsequently, we developed the extended PROMETHEE method in concordance with probabilistic simplified neutrosophic sets for multi-criteria group decision problems in which the assessments of alternatives with respect to criteria are taken as PSNSs and the information of weight of criteria is determined based on correlations among criteria. The extended PROMETHEE method not only takes into consideration probabilistic simplified neutrosophic preferences but also considers different weights of criteria for each alternative. For this reason, the method presents us wider perspective and strong theory. Furthermore, we developed a method to determine weight of decision makers by considering positive and negative preference value of each decision maker in process. In decision process, criteria weights may be changed with respect to each alternative. With this mind, we presented a method that can calculate relative weight of each criterion according to the alternatives. Finally, to demonstrate validity and reliability of the proposed method, we applied the developed method to the renewable energy resources ranking problem with probabilistic simplified neutrosophic knowledge.

Other applications of this set theory such as decision making, pattern recognition, medical diagnosis and clustering analysis are obvious questions for the future.

Compliance with ethical standards

Conflict of interest We declare that we do not have any commercial or associative interests that represent a conflict of interests in connection with this manuscript. There are no professional or other personal interests that can inappropriately influence our submitted work.

Research involving human participants and/or animals This article does not contain any studies with human participants or animals performed by any of the authors.

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