



A novel decision-making approach based on regret theory under bipolar Z-number information

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Abstract

This study introduces a new set theory called bipolar fuzzy Z-number set that makes it possible to express an information both bipolar from positive and negative perspectives, and with the reliability of it. Then, besides basic set-theoretical operations and the score function, a new distance measure is also presented. Furthermore, a decision method integrating with regret theory is proposed. Additionally, to compute the weights of criteria under bipolar Z-number information, a method in which objective and subjective criterion weights are considered simultaneously is introduced by combining the weights found by SWARA and MEREC. Also, a model with standard deviation is developed to obtain decision maker's weights. Finally, the problem of selection of paper raw material is handled to demonstrate stability and advantage of the proposed method.

Keywords Bipolar · Z-Numbers · Regret Theory · MCDM · MEREC · SWARA

1 Introduction

1.1 Literature review

Handling uncertainty and ambiguity beyond the capacity of classical set theory led to the development of several advanced uncertainty models. To address uncertainty and ambiguity in situations where classical set theory is inadequate, Fuzzy sets [55] are introduced as an important concept, providing a more flexible and expressive framework for modeling imprecise information. However, since fuzzy sets alone cannot fully capture all forms of uncertainty and ambiguity, several alternative and

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extended set theories are subsequently developed. A few of these are intuitive fuzzy sets [11], hesitant fuzzy sets [50], bipolar fuzzy sets [57], Neutrosophic sets [47], Z-number sets [56] and so on. These structures aim to provide richer semantic representation for uncertain, imprecise or conflicting information. Among these, bipolar fuzzy sets offer the opportunity to express information bipolarly and bidirectionally and allow to modeling both positive and negative opinions about an item, making them a useful method in handling situations which include both acceptance and refusal. Bipolar fuzzy sets allow the simultaneous modeling of positive and negative preferences, which makes them suitable for decision contexts where acceptance and rejection attitudes together. However, bipolar fuzzy sets do not incorporate reliability information, which is critical when expert evaluations vary in credibility. In this context, the concept of Z-number offers an effective approach for capturing the reliability of information. Z-numbers integrate two essential components: a constraint on the value and a measure of reliability associated with that value. This provides an advantage when the reliability of information is as important as the information itself. Thus, for many disciplines where reliability and risk analysis are of serious importance, it is a convenient model. Nevertheless, existing Z-number MCDM studies [32, 45, 52] generally operate in unipolar environments and do not reflect decision makers positive and negative cognitive patterns.

MCDM is a field that deals with decision-making processes in which multiple and often conflicting criteria are taken into account in evaluating and ranking alternatives. Some of the decision methods developed for complex and multi-parameter problems that we may encounter in many disciplines and social life can be summarized as: TOPSIS [25] is a method that obtain to determine the most optimal solution based on the concepts of distance to the positive ideal solution and to the negative ideal solution, respectively. If there are conflicting criteria in the decision-making process, the aim is to find the best solution by reaching a consensus, as a clear solution cannot be reached. For this purpose, VIKOR [36] which aims to find the most appropriate decision by establishing a balance between the concepts of proximity to the ideal solution and distance to the negative ideal solution is defined, ELECTRE [13] aims to reach the most appropriate result by comparing the alternatives in pairs according to the elimination system; PROMETHEE [35] aims to find the most optimal solution by determining the superiority of alternatives over each other on a criterion basis with the help of preference functions determined for each criterion; MABAC [37] allows reaching the optimum decision with the information of proximity to the boundary value determined for each criterion. These methods are extended based on different set theories [8, 30, 31, 41]. To effectively apply these methods, however, determining appropriate criterion weights is crucial, which led to growing attention to weighting methods in MCDM problems. In this context, the SWARA [28] method is widely employed to incorporate decision makers subjective judgements in a structured manner, while the MEREC [29] method is utilized as an objective weighting technique based on the removal effects of criteria. Recent studies [18, 20, 43] are applied these methods under various decision environments and application domains, demonstrating their effectiveness in capturing both preference-based and data-driven aspects of decision-making. Nevertheless, most existing studies focus on conventional information representations, leaving room for further methodological

developments. In the context of multi-criteria decision-making, just as bipolar fuzzy sets are used to address decision problems where both positive and negative preferences or evaluations must be taken into account, z -numbers can also be used to represent and manage uncertainty when making choices, especially in situations where there is a range of possible values for parameters. In summary, bipolar fuzzy sets in MCDM provide a powerful framework for dealing with complex decision problems [1, 2, 10, 18, 20, 48] where both positive and negative evaluations must be taken into account. It helps decision makers address conflicting preferences, makes more informed and sound decisions and offers more nuanced approach to evaluating and ranking alternatives based on multiple criteria. Hence bipolar fuzzy sets play an important role in decision-making scenarios where both positive and negative evaluations or preferences are required to capture the complexity and uncertainty of the decision context. Numerous studies related to application of bipolar fuzzy sets are available in the literature: An extended VIKOR method is introduced based on the bipolar fuzzy information [7], Bipolar fuzzy TOPSIS and bipolar fuzzy ELECTRE methods are presented in [6]. In [3, 4], PROMETHEE method is extended under bipolar fuzzy information. In [22], water resource management problem is handled under bipolar fuzzy environment. In [49], an extended MULTIMOORA method is introduced with bipolar fuzzy sets. In [46], bipolar fuzzy ELECTRE II is presented. In [9], a new fuzzy measure is defined based on the bipolar fuzzy information. In [21], a decision-making method is proposed to select the best treatments for COVID 19 under bipolar fuzzy environment. In [3, 4], with bipolar fuzzy information, TOPSIS and ELECTRE I methods are used for diagnosis process. Also, Z -numbers are often applied to decision-making in various fields, including engineering, risk analysis and optimization. In [33], a hybrid MCDM technique is introduced for z numbers. In [40], Z -numbers are applied for the selection of green suppliers; In [15], based on the Z -numbers, extended COPRAS method is proposed to select renewable energy resources; In [27], fuzzy MCDM method is proposed under Z -number information for the paver selection; In [42], AHP method is extended under Z -number environment for the risk analysis problem. In [23], SWARA method is applied under Z -number information. In [51], an operator and similarity measure are defined for Z -numbers. In [38], the concepts of module, inner product, cosine measure and projection measure are defined.

Classical MCDM methods such as TOPSIS, VIKOR, SWARA and MEREC assume fully rational decision makers. To address the psychological components of real-world decisions, behavioral models such as regret theory are integrated into MCDM [14, 26, 44, 54]. Regret-based TOPSIS [19], VIKOR [53] and hybrid models [58] are proposed in recent years. However, these behavioral models neither incorporate bipolar structures nor utilize Z -number reliability information.

Review of the literature reveals that bipolar fuzzy sets effectively model dual attitudes, Z -numbers effectively model reliability and credibility and regret theory effectively represents decision makers behavioral tendencies. But none of the existing study integrates these three dimensions into a unified MCDM model. Therefore, this study fills the gap by defining the bipolar Z -number set as a new uncertainty structure, introducing a regret theory-based MCDM model operating on this new set and demonstrating its applicability and interpretability within MCDM environments.

1.2 Motivation

In order to model the uncertainty in a more comprehensive dimension, which cannot be find the existing literature given above, it is desired to structure a set theory that consider membership as polar and assigns a degree of reliability to it. Therefore, due to the data obtained from this perspective include more information in any decision problem, the process can be carried out more reliably. In addition, by taking into account expert's predictions during a decision process and the emotions they experience in case the outcome is negative, we can better understand how individuals make decisions under uncertainty. With this though, in the decision-making method developed based on the novel data type mentioned above, it is aimed to develop a model by including the behavioral psychology of experts in the process. Thus, new set theory and regret theory are integrated. Also, unlike other studies in the literature, we consider objective and subjective criterion weights simultaneously. To eliminate these gaps, a novel method based on regret theory under bipolar Z-number environment is proposed.

The main contributions of the study are concluded as follows:

- Decision makers can take a more comprehensive and risk aware approach by using bipolar Z-numbers in decision-making. They can consider a wider range of scenarios, make more sound decisions and be better prepared to deal with uncertainty and variability in their choices.
- Unlike existing studies, basic operations are defined by considering the reliability information associated with Z-numbers as probability information, as it should be. Thus, the theory could be used more appropriately for its purpose.
- Based on the proposed theory, the bipolar Z-number MCDM method that handle the regret behavior of decision makers is presented.
- To determine the criterion weights, a method is developed with SWARA and MEREC, taking into account both subjective and objective perspectives.
- Using a standard deviation from the mean, a novel method is developed to get decision maker weights.

To observe the effectiveness and suitability of the proposed method, the paper raw material selection problem is addressed. Then, a comparative analysis is presented to show how effective it is.

The rest of the paper is organized as follows:

In Sect. 2, some basic background related to Z-numbers, bipolar fuzzy sets and regret theory is introduced. Section 3 presents a new set theory called bipolar Z-number set theory with some basic operations, aggregation operator and a new distance measure. Section 4 proposes a novel MCDM method including regret theory and offer to obtain criteria weights both objective and subjective utilizing SWARA and MEREC methods. Also, standard deviation method is used to calculate decision makers weights under bipolar Z-number environment. In Sect. 5, the proposed method is applied to a paper raw selection problem. Section 6 provides comparative analysis with existing methods and presents a sensitivity analysis, Sect. 7 presents a sensitivity analysis and in the reminder of the paper, the results

are presented and discussed, followed by an outline of potential directions for the future.

1.3 Research gap

Although several studies published after 2022 advanced the theories of bipolar fuzzy sets and Z-numbers independently, the existing literature still lacks a unified framework that integrates bipolar attitudes with Z-number-based reliability modeling. Recent studies interested in bipolar fuzzy environments for MCDM problems and also investigated applications of Z-numbers. However, these two concepts are not combined to represent both positive and negative evaluations and their reliability within a single structure.

Additionally, while MEREC method and its post 2022 extensions shown strong performance in objective weighting, any there is any study to examine the applicability of bipolar Z-number structure. The absence of such integration limits the ability to model uncertain, conflicting and reliability dependent decision models.

Moreover, although regret-based approaches are widely recognized for improving decision stability by minimizing relative loss, their integration with bipolar Z-number evaluations is not explored, systematically. As a result, there is a clear need for a unified model that simultaneously addresses uncertainty, conflicting evaluations, reliability and regret minimization, providing more robust and realistic MCDM methods.

This study aims to fill these gaps by proposing a comprehensive decision-making model capable of representing dual-attitude evaluations together with reliability parameter.

2 Preliminaries

Definition 2.1 [56] A Z-number is an ordered pair of fuzzy numbers denoted as $Z = (A, B)$ where first component A is fuzzy restriction on the values X and B is the reliability for the restriction A .

Definition 2.2 [57] Let X universe of discourse. Then, a bipolar fuzzy set is defined as A

$$A = \{ \langle x, (\mu_A^+(x), \mu_A^-(x)) \rangle : x \in X \} \quad (2.1)$$

where the degree of membership is defined as bipolar as $\mu_A^+(x)$ and $\mu_A^-(x)$ under the condition $\mu_A^+(x) \in [0, 1]$ and $\mu_A^-(x) \in [-1, 0]$.

Briefly, a bipolar fuzzy number is denoted as (μ_A^+, μ_A^-) where $\mu_A^+ \in [0, 1]$ and $\mu_A^- \in [-1, 0]$.

Definition 2.3 [24] Let $\alpha_1 = (\mu_1^+, \mu_1^-)$ and $\alpha_2 = (\mu_2^+, \mu_2^-)$ be two bipolar fuzzy number. Then,

1. $\alpha_1 \otimes \alpha_2 = (\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+, -\mu_1^- \mu_2^-)$;
2. $\alpha_1 \otimes \alpha_2 = (\mu_1^+ \mu_2^+, \mu_1^- + \mu_2^- - \mu_1^- \mu_2^-)$;
3. $k\alpha_1 = (1 - (1 - \mu_1^+)^k, -(-\mu_1^-)^k)$;
4. $(\alpha_1)^k = ((\mu_1^+)^k, -1 + (1 + \mu_1^-)^k)$

There are many principles that incorporate human behavioral parameters to model decision-making under uncertainty. The idea of processing the degrees of satisfaction and regret that may occur as a result of the decision to the process is the philosophy underlying the theory of regret. In many cases, human behavior contradicts utility theory, as in the famous paradox posed by Allais [5]. Regret theory is presented by including regret parameter to the utility theory. The expectation of completing the process with the least regret is a natural desire. In the regret theory, which is first defined by [12] and [34] at the same time, the level of regret that can be felt after the process is taken into account. Technically, this theory works as follow:

Let A_1 and A_2 be two possible situations. a_1 and a_2 be the outcomes that occur when A_1 and A_2 are selected, respectively. Then, the perceived utility for situation A_1 is

$$u(a_1, a_2) = v(a_1) + R(v(a_1) - v(a_2)) \quad (2.2)$$

where $v(a_1)$ and $v(a_2)$ are utility value of situations A_1 and A_2 , respectively. $R(v(a_1) - v(a_2))$ represents the regret and satisfaction value of decision maker only with result a_1 without result a_2 .

If $R(v(a_1) - v(a_2)) < 0$, then it is the regret value;

If $R(v(a_1) - v(a_2)) > 0$, then it is satisfaction value.

Generally, it is taken as $v(a) = a^\alpha$ and $R(\Delta v) = 1 - e^{-\delta \Delta v}$. Here, α is the risk aversion coefficient and $0 < \alpha < 1$. The higher α value means lower risk aversion. δ is regret avoidance coefficient with $\delta > 0$ and the larger δ , the higher the decision maker's avoidance of regret. Also it is clear that $v(a)$ and $R(\Delta v)$ are monotonic and concave functions with $v'(a) > 0$, $v''(a) < 0$, $v(0) = 0$ and $R'(\Delta v) > 0$, $R''(\Delta v) < 0$, $R(0) = 0$. Also if $\Delta v \leq 0$, then $R(\Delta v) \leq 0$ and as $|\Delta v|$ gets bigger, Δv bigger too. The larger the difference in outcomes between states A_1 and A_2 , the greater will be the perceptual regret of choosing state A_1 .

3 Bipolar Z-number sets

Definition 3.1 Let X universe of discourse and $B \subset X$. A bipolar Z-number set on X is defined as $B = \{ \langle x, (\mu_B^+(x), \mu_B^-(x)), p_B(x) \rangle : x \in X \}$ where $p_B(x)$ is a number which indicates the reliability of $(\mu_B^+(x), \mu_B^-(x))$ and $(\mu_B^+(x), \mu_B^-(x))$ is the bipolar number where $\mu_B^+(x) \in [0, 1]$ and $\mu_B^-(x) \in [-1, 0]$. The reliability degree $p_B(x)$ of

$(\mu_B^+(x), \mu_B^-(x))$ is the value between 0 and 1. Through the paper, a bipolar Z-number (BZN) is represented briefly as $((\mu_B^+, \mu_B^-), p_B)$.

In this study, the reliability component $p_B(x)$ is initially defined as a precise value in the interval $[0,1]$, representing the confidence level of the provided information. However, during the aggregation and computational process, a modified calculation technique is employed. This technique is specially designed to prevent the reliability values from converging toward near-zero levels when a large amount of information is involved, which could otherwise lead to numerically insignificant or misleading results. Therefore, the proposed transformation preserves the interpretability of the reliability measure while maintaining computational stability.

Definition 3.2 Let $b_1 = ((\mu_1^+, \mu_1^-), p_1)$ and $b_2 = ((\mu_2^+, \mu_2^-), p_2)$ be two BFZNs. Then, the operational laws are defined as below:

1. $b_1 \oplus b_2 = ((\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+, -(-\mu_1^-)(-\mu_2^-)), 2! \frac{p_1 p_2}{p_1 + p_2})$;
2. $b_1 \otimes b_2 = ((\mu_1^+ \mu_2^+, -(-\mu_1^- - \mu_2^- - \mu_1^- \mu_2^-)), 2! \frac{p_1 p_2}{p_1 + p_2})$;
3. $\lambda b_1 = ((1 - (1 - \mu_1^+)^{\lambda}, -(-\mu_1^-)^{\lambda}), p_1), \lambda > 0$;
4. $b_1^{\lambda} = (((\mu_1^+)^{\lambda}, -(1 - (1 - (-\mu_1^-))^{\lambda})), p_1), \lambda > 0$;

Remark 3.1 The operational laws introduced in Definition 3.2 provide the fundamental algebraic structure required to manipulate bipolar Z-numbers within the proposed decision-making framework. These operations enable the combination of positive and negative membership degrees along with reliability information in a mathematically consistent manner. In practical terms, these operations are used in later sections to construct aggregation procedures, generate compositive evaluations and support the bipolar reasoning mechanism. Without these operational rules, weighting and ranking process could not be implemented.

Theorem 3.3 Let $b_1 = ((\mu_1^+, \mu_1^-), p_1)$ and $b_2 = ((\mu_2^+, \mu_2^-), p_2)$ be two BFZNs. The four operations defined in Definition 3.2 are closed in the set of BZN.

Proof. Let $b_1 = ((\mu_1^+, \mu_1^-), p_1)$ and $b_2 = ((\mu_2^+, \mu_2^-), p_2)$ be two BFZNs where $\mu_1^+, \mu_2^+ \in [0,1]$, $\mu_1^-, \mu_2^-(x) \in [-1,0]$ and $p_1, p_2 \in [0,1]$.

- 1) For all $\mu_1^+, \mu_2^+ \in [0,1]$, $\mu_1^-, \mu_2^-(x) \in [-1,0]$ and $p_1, p_2 \in [0,1]$, we have $0 \leq \mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+ \leq 1$, $-1 \leq -(-\mu_1^-)(-\mu_2^-) \leq 0$, and $0 \leq 2! \frac{p_1 p_2}{p_1 + p_2} \leq 1$.

This satisfies bipolar membership and reliability representation.

- 2) The proof can be obtained similar way.
- 3) For all $\mu_1^+ \in [0,1]$, $\mu_1^- \in [-1,0]$ and $p_1 \in [0,1]$, we have $1 - (1 - \mu_1^+)^{\lambda} \in [0,1]$ and $-(-\mu_1^-)^{\lambda} \in [-1,0]$.

Thus, the form of bipolar Z-number is preserved.

4) The proof can be obtained similar way.

Theorem 3.4 Let $b_1, b_2, b_3 \in \text{BZNs}$ and $\lambda > 0$. Then, the following properties are provided:

- i. $b_1 \oplus b_2 = b_2 \oplus b_1$;
- ii. $(b_1 \oplus b_2) \oplus b_3 = b_1 \oplus (b_2 \oplus b_3)$;
- iii. $\lambda(b_1 \oplus b_2) = \lambda b_1 \oplus \lambda b_2$;
- iv. $b_1 \otimes b_2 = b_2 \otimes b_1$;
- v. $(b_1 \otimes b_2) \otimes b_3 = b_1 \otimes (b_2 \otimes b_3)$;
- vi. $(b_1 \otimes b_2)^\lambda = (b_1)^\lambda \otimes (b_2)^\lambda$.

Proof

i.

$$\begin{aligned} b_1 \oplus b_2 &= \left((\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+, -(-\mu_1^-)(-\mu_2^-)), 2! \frac{p_1 p_2}{p_1 + p_2} \right) \\ &= \left((\mu_2^+ + \mu_1^+ - \mu_2^+ \mu_1^+, -(-\mu_2^-)(-\mu_1^-)), 2! \frac{p_2 p_1}{p_2 + p_1} \right) = b_2 \oplus b_1. \end{aligned}$$

ii.

$$\begin{aligned} (b_1 \oplus b_2) \oplus b_3 &= \left((\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+, -(-\mu_1^-)(-\mu_2^-)), 2! \frac{p_1 p_2}{p_1 + p_2} \right) \oplus ((\mu_3^+, \mu_3^-), p_3) \\ &= \left((\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+ + \mu_3^+ - (\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+) \mu_3^+, -(-\mu_1^-)(-\mu_2^-)(-\mu_3^-)), 3! \frac{p_1 p_2 p_3}{(p_1 + p_2)(p_1 + p_2 + p_3)} \right) \\ &= \left((\mu_1^+ + (\mu_2^+ + \mu_3^+ - \mu_2^+ \mu_3^+) - \mu_1^+ (\mu_2^+ + \mu_3^+ - \mu_2^+ \mu_3^+), -(-\mu_1^-)(\mu_2^- + \mu_3^- - \mu_2^- \mu_3^-)), 3! \frac{p_1 p_2 p_3}{(p_1 + p_2)(p_1 + p_2 + p_3)} \right) = b_1 \oplus (b_2 \oplus b_3). \end{aligned}$$

iii.

$$\begin{aligned} \lambda(b_1 \oplus b_2) &= \lambda \left((\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+, -(-\mu_1^-)(-\mu_2^-)), 2! \frac{p_1 p_2}{p_1 + p_2} \right) \\ &= \left(\left(1 - (1 - (\mu_1^+ + \mu_2^+ - \mu_1^+ \mu_2^+))^\lambda, -(-(-(-\mu_1^-)(-\mu_2^-)))^\lambda \right), 2! \frac{p_1 p_2}{p_1 + p_2} \right) \\ &= \left(\left(1 - (1 - \mu_1^+)^\lambda (1 - \mu_2^+)^\lambda, -(-\mu_1^-)^\lambda (-\mu_2^-)^\lambda \right), 2! \frac{p_1 p_2}{p_1 + p_2} \right) = \lambda b_1 \oplus \lambda b_2. \end{aligned}$$

iv. It can be obtained similarly to proof (i).

v. It can be obtained similarly to proof (ii).

vi.

$$\begin{aligned}
(b_1 \otimes b_2)^\lambda &= \left((\mu_1^+ \mu_2^+, -(\mu_1^- - \mu_2^- - \mu_1^- \mu_2^-)), 2! \frac{p_1 p_2}{p_1 + p_2} \right)^\lambda \\
&= \left(\left((\mu_1^+ \mu_2^+)^\lambda, -\left(1 - (1 - (-(-\mu_1^- - \mu_2^- - \mu_1^- \mu_2^-)))^\lambda \right) \right), 2! \frac{p_1 p_2}{p_1 + p_2} \right) \\
&= \left(\left((\mu_1^+)^\lambda (\mu_2^+)^\lambda, -\left(1 - (1 + \mu_1^- + \mu_2^- + \mu_1^- \mu_2^-)^\lambda \right) \right), 2! \frac{p_1 p_2}{p_1 + p_2} \right) \\
&= \left(\left((\mu_1^+)^\lambda (\mu_2^+)^\lambda, -\left(1 - (1 - (-\mu_1^-))^\lambda (1 - (-\mu_2^-))^\lambda \right) \right), 2! \frac{p_1 p_2}{p_1 + p_2} \right) = (b_1)^\lambda \otimes (b_2)^\lambda.
\end{aligned}$$

Definition 3.3 Let $b = ((\mu_B^+(x), \mu_B^-(x)), p_B)$ be a BZN. Then, score and accuracy function is defined, respectively, as.

$$S(b) = \frac{(1 + \mu_B^+(x) + \mu_B^-(x)) \cdot p_B}{2}, A(c) = \mu_B^+(x)p_B + \mu_B^-(x)p_B \quad (3.1)$$

Remark 3.2 Definition 3.3 provides the ranking mechanism that allow bipolar Z-numbers to be ordered. Such ordering is essential for any MCDM procedure since the final ranking of alternatives depends on the ability to compare BZNs in a unified scale. This score measure is later employs in the numerical example and decision-making phase to rank alternatives based on their aggregated bipolar z-valuations. Hence, although the definition appears abstract, it plays a critical role in the implementation of the proposed method.

Definition 3.4 Let $b_1 = ((\mu_{B_1}^+(x), \mu_{B_1}^-(x)), p_{B_1})$ and $b_2 = ((\mu_{B_2}^+(x), \mu_{B_2}^-(x)), p_{B_2})$ be two BFZN. Then,

- If $S(b_1) < S(b_2)$, then b_2 is greater than b_1 , i.e., $b_1 < b_2$.
- If $S(b_1) = S(b_2)$, then
 - If $A(b_1) < A(b_2)$, then b_2 is greater than b_1 , i.e., $b_1 < b_2$.
 - If $(b_1) = A(b_2)$, then b_2 equal to b_1 , i.e., $b_1 \approx b_2$.

Remark 3.3 Although the lexicographic order gives priority to the score function, this does not create inconsistency in the ranking of bipolar Z-numbers. The score value reflects the overall bipolar evaluation, while the reliability term serves as secondary discriminators to avoid ranking bias. Such lexicographic comparison strategies are widely used in fuzzy, intuitionistic fuzzy and Z-number environments to ensure stable and interpretable ordering.

Definition 3.5 Let $b_j = ((\mu_j^+, \mu_j^-), p_j)$ ($j = 1, 2, \dots, n$) be the collection of bipolar z- number. The aggregated value obtained by using bipolar Z-number weighted average (BZWA) operator is defined as.

$$BZWA(b_1, b_2, \dots, b_n) = \sum_{j=1}^n w_j b_j = w_1 b_1 \oplus w_1 b_1 \oplus \dots \oplus w_n b_n$$

$$= \left(\left(1 - \prod_{j=1}^n (1 - \mu_j^+)^{w_j}, - \prod_{j=1}^n (-\mu_j^-)^{w_j} \right), n! \frac{\prod_{j=1}^n p_j}{\prod_{k=2}^n (\sum_{j=1}^k p_j)} \right) \quad (3.2)$$

Where $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

If $w_j = \frac{1}{n}$ for all j , BZWA operator is induced to the bipolar Z-number arithmetic aggregation (BZAA) operator.

Remark 3.4 The operational laws defined in Definition 3.2 already incorporate a specially constructed addition and multiplication mechanism for BZNs, which prevents the reliability component from shrinking to excessively small values under repeated binary framework. The BZWA operator, however, is conceptually different. It is introduced as a weighted mean-type aggregation operator for combining more than two BZNs at once. For this reason, the reliability aggregation in BZWA is not derived from the binary operations in Definition 3.2, but defined independently to reflect the averaging behavior required in multi-element aggregation. This independence does not contradict the BZN structure; rather, it reflects the distinct mathematical purpose of the BZWA operator compared to the binary interaction laws in Definition 3.2.

Definition 3.6 Let $b_j = ((\mu_j^+, \mu_j^-), p_j)$ ($j = 1, 2, \dots, n$) be the collection of bipolar z- number. The aggregated value obtained by using bipolar Z-number weighted geometric average (BZWGA) operator is defined as.

$$BZWGA(b_1, b_2, \dots, b_n) = \prod_{j=1}^n (b_j)^{w_j} = (b_1)^{w_1} \otimes (b_2)^{w_2} \otimes (b_n)^{w_n}$$

$$= \left(\left(\prod_{j=1}^n (\mu_j^+)^{w_j}, -1 + \prod_{j=1}^n (1 + \mu_j^-)^{w_j} \right), n! \frac{\prod_{j=1}^n p_j}{\prod_{k=2}^n (\sum_{j=1}^k p_j)} \right) \quad (3.3)$$

where $w_j \in [0, 1]$ and $\sum_{j=1}^n w_j = 1$.

Definition 3.7 Let $X = \{x_1, x_2, \dots, x_n\}$ be a universe of discourse and $B_1 = \left\{ \left((\mu_{B_1}^+(x_i), \mu_{B_1}^-(x_i)), p_{B_1}(x_i) \right) : x \in X \right\}$,

$B_2 = \left\{ \left(\left(\mu_{B_2}^+(x_i), \mu_{B_2}^-(x_i) \right), p_{B_2}(x_i) \right) : x \in X \right\}$ be two bipolar Z-number sets. Then, Euclidean distance between two set is defined as

$$d(B_1, B_2) = \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_1}^+(x_i) p_{B_1}(x_i) - \mu_{B_2}^+(x_i) p_{B_2}(x_i) \right|^2 + \left| \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_2}^-(x_i) p_{B_2}(x_i) \right|^2 \right)^{1/2} \quad (3.4)$$

Proof It can be seen easily that the measure defined Eq. (3.4) satisfies the condition of metric as below:

$$\begin{aligned} \text{(i)} \quad d(B_1, B_2) &= \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_1}^+(x_i) p_{B_1}(x_i) - \mu_{B_2}^+(x_i) p_{B_2}(x_i) \right|^2 \right. \\ &\quad \left. + \left| \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_2}^-(x_i) p_{B_2}(x_i) \right|^2 \right)^{\frac{1}{2}} \geq 0. \\ \text{(ii)} \quad \text{If } B_1 = B_2, &\text{ then } \mu_{B_1}^+(x_i) = \mu_{B_2}^+(x_i), \mu_{B_1}^-(x_i) = \mu_{B_2}^-(x_i) \text{ and } p_{B_1}(x_i) = p_{B_2}(x_i) \\ &\text{for all } i = 1, 2, \dots, n. \text{ It is obvious that} \\ d(B_1, B_2) &= \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_1}^+(x_i) p_{B_1}(x_i) - \mu_{B_2}^+(x_i) p_{B_2}(x_i) \right|^2 \right. \\ &\quad \left. + \left| \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_2}^-(x_i) p_{B_2}(x_i) \right|^2 \right)^{\frac{1}{2}} = 0 \\ \text{(iii)} \quad d(B_1, B_2) &= \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_1}^+(x_i) p_{B_1}(x_i) - \mu_{B_2}^+(x_i) p_{B_2}(x_i) \right|^2 \right. \\ &\quad \left. + \left| \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_2}^-(x_i) p_{B_2}(x_i) \right|^2 \right)^{\frac{1}{2}} \\ &= \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_2}^+(x_i) p_{B_2}(x_i) - \mu_{B_1}^+(x_i) p_{B_1}(x_i) \right|^2 \right. \\ &\quad \left. + \left| \mu_{B_2}^-(x_i) p_{B_2}(x_i) - \mu_{B_1}^-(x_i) p_{B_1}(x_i) \right|^2 \right)^{\frac{1}{2}} = d(B_2, B_1) \\ \text{(iv)} \quad d(B_1, B_2) &= \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_1}^+(x_i) p_{B_1}(x_i) - \mu_{B_2}^+(x_i) p_{B_2}(x_i) \right|^2 \right. \\ &\quad \left. + \left| \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_2}^-(x_i) p_{B_2}(x_i) \right|^2 \right)^{\frac{1}{2}} \end{aligned}$$

$$\begin{aligned}
&= \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_1}^+(x_i) p_{B_1}(x_i) - \mu_{B_2}^+(x_i) p_{B_2}(x_i) + \mu_{B_3}^+(x_i) p_{B_3}(x_i) - \mu_{B_3}^+(x_i) p_{B_3}(x_i) \right|^2 \right. \\
&\quad \left. + \left| \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_2}^-(x_i) p_{B_2}(x_i) + \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_1}^-(x_i) p_{B_1}(x_i) \right|^2 \right)^{1/2} \\
&\leq \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_1}^+(x_i) p_{B_1}(x_i) - \mu_{B_3}^+(x_i) p_{B_3}(x_i) \right|^2 + \left| \mu_2^+(x_i) p_{B_2}(x_i) - \mu_{B_3}^+(x_i) p_{B_3}(x_i) \right|^2 \right. \\
&\quad \left. + \left| \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_3}^-(x_i) p_{B_3}(x_i) \right|^2 + \left| \mu_{B_2}^-(x_i) p_{B_2}(x_i) - \mu_{B_1}^-(x_i) p_{B_1}(x_i) \right|^2 \right)^{1/2} \\
&\leq \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_{B_1}^+(x_i) p_{B_1}(x_i) - \mu_{B_3}^+(x_i) p_{B_3}(x_i) \right|^2 \right. \\
&\quad \left. + \left| \mu_{B_1}^-(x_i) p_{B_1}(x_i) - \mu_{B_3}^-(x_i) p_{B_3}(x_i) \right|^2 \right)^{1/2} \\
&\quad + \left(\frac{1}{2n} \sum_{i=1}^n \left| \mu_2^+(x_i) p_{B_2}(x_i) - \mu_{B_3}^+(x_i) p_{B_3}(x_i) \right|^2 + \left| \mu_{B_2}^-(x_i) p_{B_2}(x_i) - \mu_{B_3}^-(x_i) p_{B_3}(x_i) \right|^2 \right)^{1/2} \\
&= d(B_1, B_3) + d(B_2, B_3)
\end{aligned}$$

4 A novel method for decision-making under bipolar Z-number information

In many real-world multi-criteria decision-making problems, decision makers (DMs) are required to evaluate a set of alternatives $A = \{A_1, A_2, \dots, A_n\}$ with respect to multiple, often conflicting criteria $C = \{C_1, C_2, \dots, C_m\}$. However, the judgements provided by decision makers $D = \{D^1, D^2, \dots, D^k\}$ typically involve both reliability and uncertainty and may simultaneously contain positive and negative components. Such situations naturally lead to an evaluation environment characterized by bipolarity and imprecise or cognitive-based information which cannot be handled adequately by classical fuzzy or interval valued approaches. To present this complex structure, bipolar Z-numbers provide a powerful modeling tool since they incorporate the evaluation of an alternative under a criterion and the reliability of this evaluation, each represented in bipolar form. Moreover, in real decision environments, decision makers often experience regret due to comparisons among alternatives; therefore, regret theory must also be integrated into the evaluation framework.

The aim of this section is therefore to introduce a novel decision-making method operating under bipolar Z-number information, enriched by regret theory, and capable of handling uncertain, conflicting and reliability dependent assessments. The general structure of the problem and the steps of the proposed method are detailed below.

Let A_i ($i = 1, 2, \dots, n$) be alternative, C_j ($j = 1, 2, \dots, m$) be criterion and D^l ($l = 1, 2, \dots, k$) represents the decision maker in a decision problem. Developed method for decision problems under bipolar Z-number environment including the regret theory is given below steps:

Step 1 Construct the decision matrix.

The evaluation matrix of alternative A_i ($i = 1, 2, \dots, n$) according to the criterion C_j ($j = 1, 2, \dots, m$) for decision maker D^l ($l = 1, 2, \dots, k$) is represented with (b_{ij}^l) where b_{ij}^l is the form of bipolar z- number, i.e. $(b_{ij}^l) = ((\mu_{ij}^{l+}, \mu_{ij}^{l-}), p_{ij}^l)$, $\mu_{ij}^{l+}(x) \in [0, 1]$, $\mu_{ij}^{l-}(x) \in [-1, 0]$ and $p_{ij}^l \in (0, 1]$.

Step 2 Calculate the weight of decision maker γ_l ($l = 1, 2, \dots, k$) with standard deviation method using equation below

$$\gamma_l = \frac{\sigma_l}{\sum_{l=1}^k \sigma_l}, \sigma_l = \sqrt{\sum_{j=1}^m \sum_{i=1}^n (d((\widetilde{b}_{ij}^l, \widetilde{b}_j^l))^2} \quad (4.1)$$

where $(\widetilde{b}_{ij}^l)$, normalization of b_{ij}^l as follow

$$(\widetilde{b}_{ij}^l) = \begin{cases} ((\mu_{ij}^{l+}, \mu_{ij}^{l-}), p_{ij}^l) & , j \text{ is benefit criteria} \\ ((1 - \mu_{ij}^{l+}, -1 - \mu_{ij}^{l-}), 1 - p_{ij}^l) & , j \text{ is cost criteria} \end{cases} \quad (4.2)$$

and $(\widetilde{b}_j^l) = BZAA((\widetilde{b}_1^l), (\widetilde{b}_2^l), \dots, (\widetilde{b}_n^l))$ for $j = 1, 2, \dots, m$. When the benefit criterion is in question, it is likely that p increase as the reliability of the information increases; conversely, when the cost criterion is in question, the lower value is better, and reliability has negative effect. To reverse this situation, the reliability value should be taken as $1 - p$. More specially, for any cost criterion, a high confidence level indicates a high probability of an undesirable event. This indicates that the negative or undesirable event is almost certain to be true. A high p for a cost criterion is not necessarily positive. Therefore, the confidence value must be inverted as $1 - p$. Consequently, due to the nature of cost-type criteria, higher values are perceived more negatively by decision maker. Therefore, when an expert assigns a high degree of confidence p to a high-cost or undesirable value, this implies strong confidence in the accuracy of the negative information. In such a case, it is not rational for confidence to contribute positively; rather, the decision model should consider this certainty as a negative element. In line with this requirement, the reliability component is inverted for cost-type criteria using the $1 - p$ transformation.

The standard deviation value of the decision maker's evaluations represents information about how distinctive the expert is within the system. The standard deviation of expert assessments varies with the amount of distinctive information they provide. A higher standard deviation indicates a stronger to discriminate between alternatives, consistent with the information-theoretic principle used in CRITIC [17] and distributional expert weighting [16]. Thus, experts with higher standard deviation values receive higher weights, while experts with nearly uniform assessments with lower standard deviations provide less information. Therefore, experts with higher standard deviations are assigned higher weights,

while experts with lower standard deviations, that is those with nearly identical assessments, are assigned lower weights for their less contributions to the system.

Step 3 Calculate combined criteria weights w_j ($j = 1, 2, \dots, m$) by using SWARA and MEREC methods.

First, apply SWARA method for the calculation of subjective criteria weights as following steps.

Step 3.1.1 Determine the attributes C_j ($j = 1, 2, \dots, m$) obtained by experts related to the decision problem

Step 3.1.2 Determine the relative importance degree S_j which assigned by each decision maker, subjectively and subsequently obtain the values K_j where

$$K_j^l = \begin{cases} 1, & j = 1 \\ S_j + 1, & j > 1 \end{cases} \quad (4.3)$$

Step 3.1.3 Calculate proportional weights of criteria q_j with preliminary calculation as

$$q_j^l = \begin{cases} 1, & j = 1 \\ \frac{K_{j-1}}{K_j}, & j > 1 \end{cases} \quad (4.4)$$

Step 3.1.4 Obtain the final weights of criteria as

$$\xi_j^l = \frac{q_j^l}{\sum_{j=1}^m q_j^l} \quad (4.5)$$

For the calculation of objective criteria weights, MEREC method is applied as steps below.

Step 3.2.2 Calculate the overall performance of alternatives according to each decision maker using logarithmic measure as below

$$S_i^l = \ln \left(1 + \left(\frac{1}{m} \sum_j \left(\left| \ln(\tilde{b})_{ij}^l \right| \right) \right) \right) \quad (4.6)$$

Step 3.2.3 Determine the performance of alternatives by removing each criterion with the following equation

$$\hat{S}_{ij}^l = \ln \left(1 + \left(\frac{1}{m} \sum_{h, h \neq j} \left(\left| \ln(\tilde{b})_{ih}^l \right| \right) \right) \right) \quad (4.7)$$

Step 3.2.4 Calculate total absolute deviations as

$$E_j^l = \sum_i \left| \hat{S}_{ij}^l - S_i^l \right| \quad (4.8)$$

Step 3.2.5 Obtain the objective weights of criterion for each decision maker as

$$\zeta_j^l = \frac{E_j^l}{\sum_j E_j^l} \quad (4.9)$$

Finally combined criteria weights w_j^l are obtained as below:

$$w_j^l = \frac{\zeta_j^l \zeta_j^l}{\sum_{j=1}^m \zeta_j^l \zeta_j^l} \quad (4.10)$$

This multiplicative integration strengthens criteria that are robust both subjective and objective perspective. It eliminates the need for any external parameters, and it is ensured that the final weights satisfy $\sum_{j=1}^m w_j^l = 1$.

Step 4 Weighting the evaluation matrix obtained by each decision maker as

$$t^l = (t_{ij}^l) = (w_j^l b_{ij}^l) = \left(\left(1 - (1 - \mu_{ij}^{l+})^{w_j^l}, -(-\mu_{ij}^{l-})^{w_j^l} \right), p_{ij}^l \right) \quad (4.11)$$

where $t_{ij}^l = ((\mu_{ij}^{l+}, \mu_{ij}^{l-}), p_{ij}^l)$

Step 5 Determine the group decision matrix $t = (t_{ij})$

$$BZWA(t^1, t^2, \dots, t^k) = \sum_{l=1}^k \gamma_l t^l = \left(\left(1 - \prod_{l=1}^k (1 - \mu_{ij}^{l+})^{\gamma_l}, \prod_{l=1}^k (-\mu_{ij}^{l-})^{\gamma_l} \right), k! \frac{\prod_{l=1}^k p_{ij}^l}{\prod_{s=2}^k (\sum_{l=1}^s p_{ij}^l)} \right) \quad (4.12)$$

Step 6 Calculate regret matrix u^+ and rejoice matrix u^- , respectively, as

$$\begin{aligned} u^+ &= (u_{ij}^+) = v(S(t_{ij})) + R(v(S(t_{ij})) - v(S(t^*))) \\ u^- &= (u_{ij}^-) = v(S(t_{ij})) + R(v(S(t_{ij})) - v(S(t^-))) \end{aligned} \quad (4.13)$$

for each decision maker where $t^* = ((1, 0), 1)$ and $t^- = ((0, -1), 1)$

- The reference points $t^* = ((1, 0), 1)$ and $t^- = ((0, -1), 1)$ represent ideal and non-ideal bipolar Z-numbers, respectively. In the positive dimension, $\mu^+ = 1$ indicates full membership in the desired set, while in the negative dimension, $\mu^- = -1$ indicates full membership in the undesired set. A reliability degree $p = 1$ represents full certainty in both cases. Using these two extremes allows regret and rejoice values to be calculated according to the strongest theoretically possible contrast with bipolar Z-number environment. This is consistent with the principle of reference dependence of regret theory, where benefits are evaluated according to idealized criteria rather than absolute values.
- The reference points used are not specific to any decision problem because they are mathematical limits independent of the structure of the decision problem. Unlike data-driven reference points, the chosen reference points

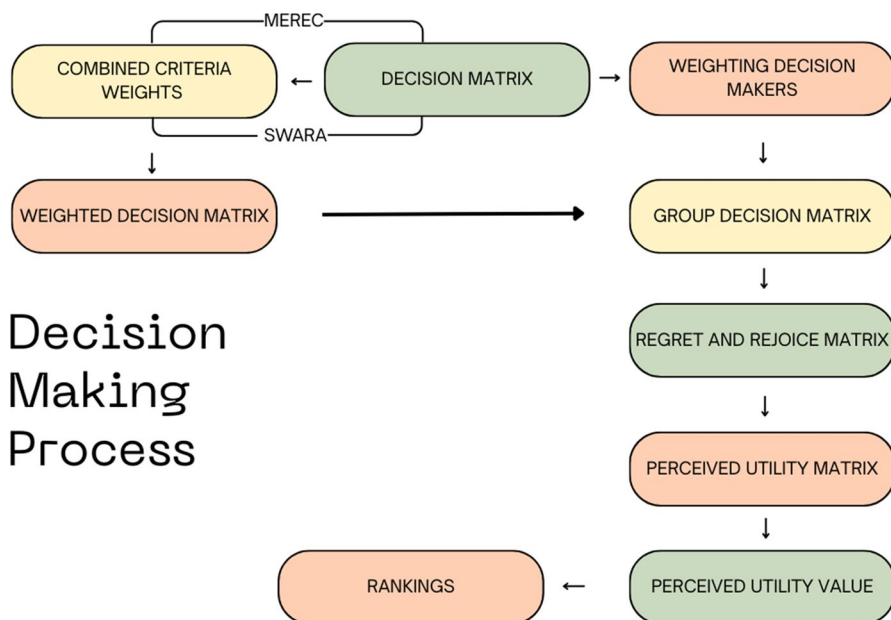


Fig. 1 Flowchart of the decision process

are model-driven and depend only on the finite nature of the bipolar Z-numbers. Since $\mu^+ \in [0,1]$, $\mu^- \in [-1,0]$ and $p = 1$, the ideal and non-ideal bipolar Z-numbers are universal across all decision-making scenarios. Therefore, these reference points are dataset independent and remain valid for any bipolar Z-number MCDM problem. These reference points can be generalized to any decision scenario involving BZN environment.

- The utility function $v(a) = a^\alpha$ and the regret function $R(\Delta v) = 1 - e^{-\delta \Delta v}$ are not problem-specific constructs; they are fundamental functions that model human behavior and are widely used in decision models within the scope of regret theory. Bipolar Z-numbers can be converted to the range $[0,1]$ via the score function and directly applied to convert these values into psychologically meaningful benefits and regret intensities. The score function reduces bipolar Z-numbers to the interval $[0,1]$. Therefore, the function $v(a) = a^\alpha$ is mathematically suitable for reshaping these scores according to risk sensitivity. The function $R(\Delta v) = 1 - e^{-\delta \Delta v}$ is the standard form of difference-based regret models. The characteristic feature of this function is that regret grows logarithmically as the difference increases, creating behavioral anxiety at a very small differences and allowing decision maker sensitivity to be adjusted by δ . On the other hand, the α and δ parameters allow the decision

Table 1 Types of criteria

Criterion	Justification	Type
Brightness	Higher brightness improves paper appearance and quality	Benefit-type
Durability	Higher durability increases mechanical strength and usability	Benefit-type
Moisture content	Excessive moisture reduces quality, increases deformation and reduces printing performance, thus lower values are preferred	Cost-type
Smoothness	Higher smoothness provides better printability and surface quality	Benefit-type

Table 2 Evaluation matrix given by decision maker D^1

	C_1	C_2	C_3	C_4
A_1	((0.55, -0.29), 0.5)	((0.5, -0.3), 0.7)	((0.11, -0.45), 0.2)	((0.5, -0.26), 0.4)
A_2	((0.12, -0.63), 0.6)	((0.72, -0.25), 0.6)	((0.04, -0.19), 0.3)	((0.78, -0.1), 0.2)
A_3	((0.1, -0.2), 0.3)	((0.21, -0.48), 0.3)	((0.58, -0.03), 0.4)	((0.16, -0.4), 0.7)
A_4	((0.11, -0.5), 0.2)	((0.17, -0.43), 0.5)	((0.32, -0.44), 0.5)	((0.06, -0.62), 0.6)
A_5	((0.03, -0.81), 0.6)	((0.39, -0.05), 0.4)	((0.57, -0.31), 0.6)	((0.67, -0.09), 0.7)

maker to adapt to different decision environments by adjusting their risk attitude and regret sensitivity.

- In our proposed method, the parameters $\alpha = 0.88$ and $\delta = 0.3$ are chosen based on standard practice in regret theory. Specially, α defines the curvature of the utility function, reflecting the diminishing sensitivity of decision makers to gains, while δ controls the sensitivity of the regret function to differences between alternatives. These values are widely adopted in the literature and ensure behaviorally consistent and optimal decision results.

Step 7 Obtain the perceived utility matrix $u = (u_{ij})$ as

$$u_{ij} = \beta u_{ij}^+ + (1 - \beta) u_{ij}^-, i = 1, 2, \dots, n; j = 1, 2, \dots, m \quad (4.14)$$

Here, $\beta \in [0, 1]$ is taken according to the degree of regret or rejoice situations.

Step 8 Calculate the perceived utility value S_i ($i = 1, 2, \dots, n, l = 1, 2, \dots, k$) for each alternative as.

$$S_i = \sum_{j=1}^m u_{ij} \quad (4.15)$$

Step 9 Determine the optimal alternative according to the S_i ($i = 1, 2, \dots, n$) values. The flowchart of this method is presented in Fig. 1.

Table 3 Evaluation matrix given by decision maker D^2

	C_1	C_2	C_3	C_4
A_1	$((0.57, -0.33), 0.3)$	$((0.6, -0.7), 0.5)$	$((0.21, -0.38), 0.3)$	$((0.5, -0.76), 0.7)$
A_2	$((0.12, -0.76), 0.2)$	$((0.62, -0.43), 0.3)$	$((0.03, -0.29), 0.6)$	$((0.3, -0.7), 0.5)$
A_3	$((0.2, -0.3), 0.6)$	$((0.2, -0.421), 0.3)$	$((0.54, -0.05), 0.5)$	$((0.26, -0.7), 0.4)$
A_4	$((0.3, -0.44), 0.7)$	$((0.27, -0.57), 0.4)$	$((0.23, -0.36), 0.2)$	$((0.07, -0.26), 0.7)$
A_5	$((0.01, -0.66), 0.4)$	$((0.44, -0.09), 0.5)$	$((0.57, -0.57), 0.5)$	$((0.26, -0.5), 0.4)$

Table 4 Combined criteria weights

	C_1	C_2	C_3	C_4
D^1	0.331	0.158	0.154	0.356
D^2	0.325	0.192	0.232	0.248

Table 5 Weighted decision matrix of D^1

	C_1	C_2	C_3	C_4
A_1	$((0.232, -0.663), 0.5)$	$((0.205, -0.671), 0.7)$	$((0.03, -0.76), 0.2)$	$((0.205, -0.639), 0.4)$
A_2	$((0.04, -0.858), 0.6)$	$((0.34, -0.63), 0.6)$	$((0.01, -0.57), 0.3)$	$((0.394, -0.466), 0.2)$
A_3	$((0.03, -0.58), 0.3)$	$((0.07, -0.78), 0.3)$	$((0.24, -0.31), 0.4)$	$((0.05, -0.73), 0.7)$
A_4	$((0.03, -0.79), 0.2)$	$((0.05, -0.75), 0.5)$	$((0.11, -0.76), 0.5)$	$((0.08, -0.85), 0.6)$
A_5	$((0.01, -0.93), 0.6)$	$((0.15, -0.37), 0.4)$	$((0.24, -0.67), 0.36)$	$((0.307, -0.45), 0.7)$

Table 6 Weighted decision matrix of D^2

	C_1	C_2	C_3	C_4
A_1	$((0.24, -0.69), 0.3)$	$((0.258, -0.89), 0.5)$	$((0.07, -0.72), 0.3)$	$((0.202, -0.91), 0.7)$
A_2	$((0.04, -0.91), 0.2)$	$((0.27, -0.75), 0.3)$	$((0.009, -0.66), 0.6)$	$((0.109, -0.89), 0.5)$
A_3	$((0.07, -0.675), 0.6)$	$((0.07, -0.75), 0.3)$	$((0.223, -0.376), 0.5)$	$((0.09, -0.89), 0.4)$
A_4	$((0.1, -0.76), 0.7)$	$((0.09, -0.83), 0.4)$	$((0.08, -0.71), 0.2)$	$((0.02, -0.64), 0.7)$
A_5	$((0.003, -0.87), 0.4)$	$((0.17, -0.456), 0.5)$	$((0.24, -0.832), 0.5)$	$((0.09, -0.79), 0.4)$

5 Numerical example

In order to test the theory given above, the paper raw material selection problem presented in a bipolar fuzzy environment is discussed [39]. Paper is made from the methods: Pulping, refining, dilution process, formation of fibers, pressurization, drying, finished product for usage. Raw materials for paper making are listed as: wood, rag, grass, bagasse and hemp ($A_i, i = 1, 2, 3, 4, 5$). These materials are evaluated in terms of brightness, durability, moisture content and smoothness

Table 7 Group decision matrix

	C_1	C_2	C_3	C_4
A_1	((0.235, -0.68), 0.37)	((0.22, -0.76), 0.58)	((0.05, -0.75), 0.24)	((0.19, -0.75), 0.5)
A_2	((0.039, -0.886), 0.3)	((0.30, -0.691), 0.4)	((0.01, -0.62), 0.4)	((0.27, -0.62), 0.28)
A_3	((0.04, -0.634), 0.4)	((0.07, -0.79), 0.3)	((0.23, -0.35), 0.44)	((0.07, -0.8), 0.5)
A_4	((0.06, -0.78), 0.31)	((0.07, -0.79), 0.44)	((0.1, -0.74), 0.28)	((0.02, -0.76), 0.64)
A_5	((0.006, -0.90), 0.48)	((0.15, -0.41), 0.44)	((0.23, -0.74), 0.54)	((0.21, -0.58), 0.5)

Table 8 Regret matrix

	C_1	C_2	C_3	C_4
A_1	-0.158	-0.107	-0.274	-0.139
A_2	-0.297	-0.124	-0.199	-0.173
A_3	-0.189	-0.257	-0.012	-0.216
A_4	-0.258	-0.225	-0.246	-0.188
A_5	-0.296	-0.06	-0.108	-0.066

Table 9 Rejoice matrix

	C_1	C_2	C_3	C_4
A_1	0.177	0.224	0.07	0.194
A_2	0.048	0.208	0.139	0.163
A_3	0.148	0.085	0.312	0.123
A_4	0.084	0.115	0.095	0.149
A_5	0.049	0.268	0.224	0.262

Table 10 Perceived utility matrix

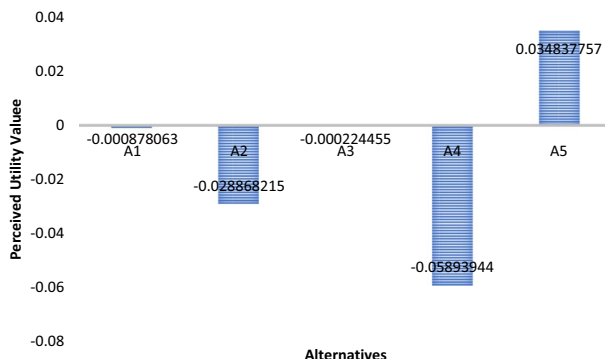
	C_1	C_2	C_3	C_4
A_1	0.009	0.058	-0.101	0.027
A_2	-0.124	0.04	-0.03	-0.005
A_3	-0.02	-0.085	0.149	-0.04
A_4	-0.087	-0.054	-0.075	-0.019
A_5	-0.123	0.103	0.058	0.098

($C_j, j = 1, 2, 3, 4$). The selection process is practiced through the proposed method in this study.

In the case study, for evaluation criteria brightness, durability, moisture content and smoothness are considered in assessing the suitability of raw materials for paper manufacturing. To ensure transparency and correctness in normalization process, each criterion is classified as either a benefit-type or cost-type attribute based on industrial standards. Brightness, durability and smoothness are benefit-type criteria

Table 11 Perceived utility value

	A_1	A_2	A_3	A_4	A_5
S_i	-0.001	-0.029	-0.0008	-0.059	0.034

**Fig. 2** Results of comparisons

since the higher values indicate better paper quality. Moisture content is a cost-type criterion since excessive moisture negatively affects the physical and printing properties of paper (Table 1).

The case study is intended to illustrate the applicability of the proposed method rather than to provide statistical generalization. Therefore, a limited number of experts are consulted. Despite the small sample size, the proposed aggregation mechanism is generic and can accommodate any number of decision makers. Extending the analysis to larger expert groups is left for future research.

The data associated with the evaluation of materials according to the parameter given above are listed below by experts:

In this study, the bipolar Z-number values used in the evaluation matrix are not derived from real industrial measurements or expert survey. Instead, they are synthetically generated for modeling and demonstration purposes. The primary objective of the case study is to illustrate the computational procedure and applicability of the proposed model under controlled conditions rather than to present empirical field data. The hypothetical bipolar values (μ^+ , μ^-) are assigned to represent typical satisfaction and dissatisfaction tendencies consistent with the pulp and paper industry. Similarly, the reliability degree p is generated synthetically within the interval $[0,1]$ to simulate varying levels of confidence. Thus, the evaluation matrix used in the example is a simulation-based modeling dataset, constructed solely to demonstrate the functioning and advantages of the proposed method.

To determine optimal material for paper raw, the steps proposed above are applied as below:

Step 1 Construct the evaluation matrix as Tables 2 and 3

Step 2 Determine the weights of decision maker using Eq. (4.1) as $\gamma_1 = 0.56$ and $\gamma_2 = 0.43$

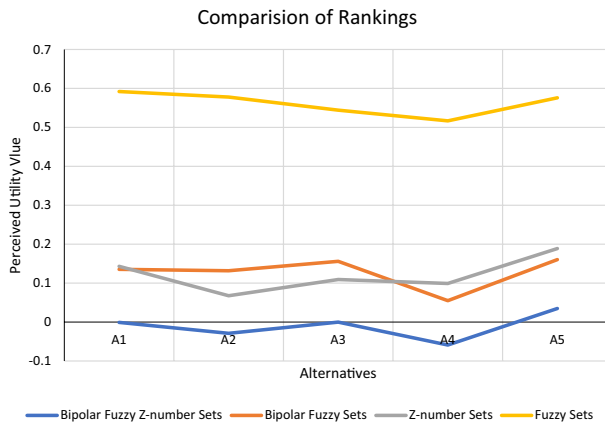


Fig. 3 Comparison of results according to the sets

Fig. 4 The results of ranking

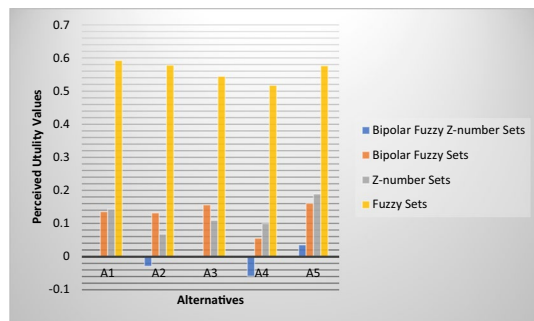


Table 12 Ranking results according to the different theories

	Ranking results	Best alternative	Worst alternative
Proposed set (BFZN)	$A_5 > A_3 > A_1 > A_2 > A_4$	A_5	A_4
Bipolar Fuzzy [57]	$A_5 > A_3 > A_1 > A_2 > A_4$	A_5	A_4
Z-number [56]	$A_5 > A_1 > A_3 > A_4 > A_2$	A_5	A_2
Fuzzy Set [55]	$A_1 > A_2 > A_5 > A_3 > A_4$	A_1	A_4

Step 3 Calculate the combined criteria weights for each decision maker as Table 4

Step 4 Obtain weighted evaluation matrix $t^l = (w_j^l b_{ij}^l)$ for each decision maker using Eq. 4.11 as Tables 5 and 6.

To see the weighting process more clearly, let's calculate how only the first alternative is weighted according to the first criterion related to the decision maker D^1 and D^2 , respectively, as follow:

$$t^1 = w_1^1 b_{11}^1 \text{ where } w_1^1 = 0.331 \text{ according to Table 3;}$$

$$\begin{aligned}
w_1^1 b_{11}^1 &= 0.331((0.55, -0.29), 0.5) = ((1 - (1 - 0.55)^{0.331}, -(-(-0.29))^{0.331}), 0.5) \\
&= ((0.232, -0.663), 0.5) \quad \text{where } w_1^2 = 0.325 \text{ according to Table 3;} \\
w_1^2 b_{11}^2 &= 0.325((0.57, -0.33), 0.3) = ((1 - (1 - 0.57)^{0.325}, -(-(-0.33))^{0.3251}), 0.5) \\
&= ((0.24, -0.69), 0.3).
\end{aligned}$$

Weighted decision matrices for each decision maker's evaluations are calculated similarly for each alternative and criterion. The results obtained for all values are given in Tables 4 and 5.

Step 5 Obtain the group decision matrix as Table 7

Step 6 Calculate regret and rejoice matrix, respectively, as Tables 8 and 9 where the risk and regret aversion coefficients are taken as $\alpha = 0.88$ and $\delta = 0.3$, respectively.

Step 7 Obtain the perceived utility matrix as Table 10

Step 8 Calculate perceived utility value for each alternative as Table 11

Step 9 Rank the alternatives according to S_i values as $A_5 > A_3 > A_1 > A_2 > A_4$.

The results can be seen also in Fig. 2

6 Comparison

Modeling uncertainty is crucial for a variety of reasons in a variety of fields and disciplines. Uncertainty is inherent in many real-world situations and it is crucial that this is acknowledged and accounted for in models and decision-making processes. Decision-making often involves predicting future outcomes or selecting actions based on current information. Models that take into account provide decision makers with more realistic expectations and provide a better understanding of the potential risks associated with different choices. In this study, a set theory that contains more information than the set theories given in the literature is defined and a decision model is developed in order to model uncertainty better. Accordingly, the results obtained with the method developed in this study on a selection problem are compared with the results obtained on other clusters.

There are five alternatives and four criteria in the problem addressed in this study. First, we handle the problem under bipolar fuzzy environment and we get the ranking as $A_5 > A_3 > A_1 > A_2 > A_4$. Then, by disabling the polarity same problem is considered with Z-number information and the results are $A_5 > A_1 > A_3 > A_4 > A_2$. Also, eliminating both polarity and reliability information, if we continue with classical fuzzy information, the results are as $A_1 > A_2 > A_5 > A_3 > A_4$. The comparison results according to the sets can be observed in Figs. 3 and 4 and Table 12.

Although the results are consistent, there are partial differences. There are some reasons behind this: Unlike others, the degree of reliability is treated as probability information, not fuzzy information and also, individuals can further weight the advantages and disadvantages of each option, taking into account both positive and negative attributes. The reason for the difference, especially when reliability information is included, is that the developed set operators provide sensitive and realistic results compared to those used in existing studies.

Table 13 Sensitive analysis of ranking results according to α and δ values

Scenarios	Ranking
$\alpha = 0.5$ and $\delta = 0.1$	$A_5 > A_1 > A_3 > A_2 > A_4$
$\alpha = 0.5$ and $\delta = 0.5$	$A_5 > A_1 > A_3 > A_2 > A_4$
$\alpha = 0.5$ and $\delta = 0.7$	$A_5 > A_1 > A_3 > A_2 > A_4$
$\alpha = 0.7$ and $\delta = 0.1$	$A_5 > A_1 > A_3 > A_2 > A_4$
$\alpha = 0.7$ and $\delta = 0.5$	$A_5 > A_1 > A_3 > A_2 > A_4$
$\alpha = 0.7$ and $\delta = 0.7$	$A_5 > A_1 > A_3 > A_2 > A_4$
$\alpha = 0.9$ and $\delta = 0.1$	$A_5 > A_3 > A_1 > A_2 > A_4$
$\alpha = 0.9$ and $\delta = 0.5$	$A_5 > A_3 > A_1 > A_2 > A_4$
$\alpha = 0.9$ and $\delta = 0.7$	$A_5 > A_3 > A_1 > A_2 > A_4$

The comparison results indicate that the proposed BZN-based method produces ranking outcomes that are generally consistent with bipolar fuzzy approaches, while exhibiting notable differences when compared with Z-number and classical fuzzy-based methods. The similarity between the proposed method and bipolar fuzzy results confirms that the bipolar structure is properly preserved. However, the observed differences, particularly in relative positions of the alternatives A_1 and A_3 , can be attributed to explicit modeling of reliability information and incorporation of regret–rejoice theory. Unlike classical Z-number approaches, the proposed method separately evaluates positive and negative information, which leads to different dominance relations among alternatives. Furthermore, the significant discrepancy with classical fuzzy set results highlights the limitations of ignoring reliability and psychological comparison effects. Overall, these differences demonstrate that the proposed method provides a more discriminative and realistic ranking mechanism.

If the data are larger, noticeable differences in the results can be observed with the precise calculations suggested in this study regarding reliability information.

The advantages of this paper stated as below:

- With the set theory proposed in this study, decision makers can simultaneously evaluate potential risks and benefits by making choices that consider not only the likelihood of positive outcomes but also potential negative outcomes. Also, by including reliability information, it allows decision makers to express uncer-

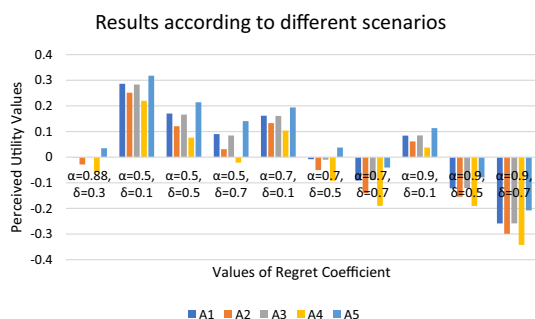
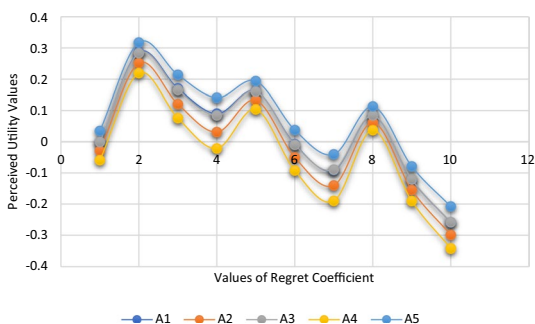
Fig. 5 Parameter sensitivity curve

Fig. 6 Distribution of regret coefficients

tainty and ambiguity in each criterion, leading to a more comprehensive evaluation of alternatives.

- By developing a model that incorporates regret theory, it is possible to better capture the dynamics of human behavior and provide more accurate predictions of decision outcomes by taking into account the emotional aspects of decision-making.
- By using the hybrid model that integrates both objective and subjective weights, the opportunity to benefit from the strengths of both types of weights is provided. It also considers the rigor of data-driven analysis and the insight and flexibility provided by expert opinion.

7 Sensitivity analysis

Ranking results to the choice of parameters α (risk aversion coefficient) and δ (regret weight), we conducted a comprehensive sensitivity analysis. Here, α represents the decision maker's degree of risk aversion in a regret theory framework (where $\alpha = 1$ implies risk neutrality, $\alpha < 1$ indicates risk aversion for gains and risk-seeking for losses, with lower values reflecting higher sensitivity to losses). The parameter δ , on the other hand, captures the weight assigned to regret (or the degree of regret aversion), where higher values of δ imply stronger consideration of potential regret in the decision process ($\delta = 1$ means full regret minimization, while lower values reduce its influence).

We tested a wide range of values for $\alpha \in \{0.5, 0.7, 0.9\}$ as covering moderate to high risk and loss aversion and $\delta \in \{0.1, 0.5, 0.7\}$ as ranking from low to high regret weighting. The resulting rankings for the alternatives A_1 to A_5 as presented below (Table 13):

The results demonstrate remarkable stability of the proposed method across most of the parameter values. In 6 out of 9 tested combinations (for $\alpha \leq 0.7$), the ranking remains identical: A_5 is consistently the top ranked alternative, followed by A_1 , A_3 , A_2 and A_4 . This full ranking is preserved regardless of the regret weight δ , indicating that variations in regret aversion have negligible impact when risk or loss aversion is moderate.

A minor change occurs only when α approaches 0.9: Here, A_1 and A_3 switch positions ($A_5 > A_3 > A_1 > A_2 > A_4$), while the relative positions of all other alternatives, including the clear dominance of A_5 and the last place of A_1 , remain unchanged. Notably, even in this case, the ranking is insensitive to δ , as the same switched order holds across all three values of δ .

These findings strongly support the robustness of the method. The top alternative A_5 is invariant across the entire tested range and the overall ranking structure is highly stable, with only a subtle interchange between the second and third positions occurring at the boundary of near-risk neutral behavior. Given that real-world decision makers typically exhibit significant loss aversion (corresponding to α values around 0.88 or lower, as commonly reported in the regret theory literature), the proposed method yields consistent and reliable rankings in practically relevant scenarios. Thus, the sensitivity analysis confirms the stability of the results and the method's practical applicability (Figs. 5 and 6).

8 Conclusion

This study introduces a novel decision-making framework based on bipolar Z-numbers, providing an integrated structure that simultaneously captures satisfaction, dissatisfaction and the associated reliability of evaluations. The main contribution lies in extending classical Z-number theory into a bipolar environment and establishing a normalization and aggregation mechanism that preserves the polarity of information. Compared with existing bipolar fuzzy or standard Z-number approaches, the proposed framework allows more effective modeling of uncertainty and conflicting attitudes while maintaining practical computability.

The case study demonstrates the applicability and interpretability of the method in MCDM problems. By incorporating reliability into bipolar assessments, the method strengthens stability, reduces information loss and supports more transparent decision process. The theoretical and methodological innovations developed in this study contribute to the wider literature on fuzzy decision-making, offering a robust foundation for modeling dual-attitude evaluations under uncertainty.

9 Discussion

9.1 Justification for interpreting the confidence component of bipolar Z-numbers as probabilistic information

- In classical Z-number set theory, reliability is often treated as an imprecise linguistic definition, e.g., “extremely credible,” “moderately credible.” However, the reliability of a judgement does not define the object, but rather the validity of the expert's own assessment. Therefore, the reliability refers to people's opinions about the truth value of statements about knowledge. It is outside the bipolar membership function and independent from positive and negative directions. Since reliability reflects an expert's certainty and precision is essentially a proba-

bilistic concept, the probabilistic interpretation is more natural and theoretically grounded, while the bipolar fuzzy part captures the uncertainty and the probabilistic reliability controls the degree of confidence in the evaluation. The degree of confidence is not a descriptive attribute of the alternative, but rather a meta-level measure of confidence that expresses the reliability of the expert's assessment.

- In classical Z-number structures, the confidence is typically represented as a fuzzy assessment expressing uncertain confidence in the expert's judgement. However, in the proposed model, the confidence rating p is treated as a probabilistic value. This choice is theoretically justified by the requirements imposed by the behavioral decision-making theories especially regret theory on which the proposed method is based. Behavioral models such as regret theory, prospect theory and utility theory are based on numerical transformations of evaluations thorough utility and regret functions as $v(a) = a^\alpha$ or $R(\Delta v) = 1 - e^{-\delta \Delta v}$. If the degree of reliability is treated as fuzzy information, it means that the reliability itself is uncertain, which add a second-order fuzziness to the model. This produces fuzzy utility and fuzzy regret values, making the regret–rejoice transformation mathematically inoperable. Probability information is preferred to ensure compliance with behavioral theory.
- In bipolar Z-number assessment, a single reliability value is given for positive and negative aspects. Reliability is the evaluator's belief in his/her own statement. It is independent of the positive and negative directions. Furthermore, bipolar environments already contain binary information, and defining separate reliabilities for positive and negative directions is reduced interpretability. A single probabilistic reliability parameter provides conceptual simplicity and preserves the internal consistency of bipolar constructs by reliability to the expert's confidence rather than the polarity of the evaluation. Probabilistic reliability provides a measurable, normalizable and behaviorally interpretable certainty value that seamlessly integrates with both subjective (SWARA) and objective (MEREC) weighting mechanism, as well as regret-based utility transformations. For these reasons, probabilistic reliability is not only mathematically more consistent, but also theoretically indispensable for ensuring compatibility with the behavioral basis of the proposed decision model.

9.2 Specificity of the euclidean distance for bipolar Z-numbers

- The Euclidean metric preserves polarity by calculating the contribution of positive and negative membership degrees separately. Furthermore, by including the reliability index as a third axis, the metric reflects not only the bipolar evaluations but also the reliability of these evaluations. Therefore, the distance measure used in Definition 3.6 is specially defined to take into account the geometric structure of bipolar Z-numbers and to exploit their multidimensional, bipolar and reliability-enhanced structure.
- Bipolar Z-numbers constitute a multidimensional information structure with positive, negative and reliability information. Because changes in one of these parameters do not affect the other, these three components are independent of each other and can be thought of as a three-axis information space. The most nat-

ural distance measure that can be defined with these independent data structures is the Euclidean distance measure.

10 Future work

The numerical example in this study is intentionally kept brief to clearly illustrate the operational steps of the proposed bipolar Z-number and regret-based decision framework. While the sample size is limited, the method itself imposes no structural restrictions on the number of criteria and alternatives and its computational complexity scales linearly with the problem size.

Future research will focus on several specific and technically applicable perspectives as:

- Applying the model to larger and more diverse real-world datasets to further demonstrate its generalizability;
- Testing the framework in different decision domains, such as energy planning, supply chain management or health assessment,
- Incorporating alternative behavioral models or adaptive reference point mechanism,
- Conducting studies with larger expert groups to increase experimental validation.
- Extending the bipolar Z-number MCDM model by integrating it with prospect theory to capture decision maker's risk preferences more comprehensively. This will allow a comparison between regret-based approaches within the a-number framework. Such an extension will enhance the applicability of the method to more complex decision scenarios and provide further insights into the behavioral aspects of decision-making under certainty.
- Developing a novel distance or similarity measures specially proposed for bipolar Z-numbers, improving the accuracy of clustering and rankings.
- Integrating the bipolar Z-number framework with machine learning techniques such as clustering, classification or reinforcement learning to enhance decision support capabilities.

Author contributions RŞ contributed to formal analysis, conceptualization, and drafting—original manuscript. AK contributed to coding and implementation. FA contributed to reviewing, conceptualization and editing original manuscript. All authors reviewed the manuscript.

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Data availability No datasets were generated or analyzed during the current study.

Declarations

Conflict of interest The authors declare no competing interests.

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